MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND

REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

FROM NOVEMBER 1910 *TO NOVEMBER* 1911.

VOL. LXXI.

LONDON:
ROYAL ASTRONOMICAL SOCIETY
BURLINGTON HOUSE, W.
1911.

PRINTED BY NEILL AND CO., LTD., EDINBURGH.

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ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXI. NOVEMBER 11, 1910. NO. 1

SIR DAVID GILL, K.C.B., F.R.S., PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Penry Vaughan Bevan, M.A., Professor of Physics, Royal Holloway College, Englefield Green (proposed by H. F. Newall);

David Brunt, B.A., Trinity College, Cambridge (proposed by H. F. Newall);

Rev. John Crouch, President of the Philosophical Institution, Paisley (proposed by Sir David Gill);

Charles Eddie, Examiner of Masters and Mates, Port of Vancouver, British Columbia (proposed by L. A. Demers);

Henry Hart, Mangalore, South Canara, India (proposed by S. G. Burrard);

Capt. Alexander Inglis, Harbour Master, Port Adelaide, Glanville, South Australia (proposed by R. W. Chapman);

Capt. W. E. Maddocks, Harbour Master, Port Swettenham, Selangor, Federated Malay States (proposed by Peter Thompson);

Capt. Francis Crofton Molesworth, R.E., United Service Club, Simla, India (proposed by J. W. Meares);

Walter Moss, Solar Physics Observatory, South Kensington, and 1 Braemar Avenue, Wimbledon Park (proposed by W. J. S. Lockyer);

John William Nicholson, M.A., D.Sc., Trinity College, Cambridge (proposed by H. F. Newall);

James Harry Smith, F.C.S., M.P.S., 5 Turks Road, Radcliffe, Lancashire (proposed by George Innes);

Morris Francis Weinrich, M.E., A.M,, Ph.D., Instructor in Physics, Stevens Institute, Hoboken, New Jersey, U.S.A. (proposed by S. A. Mitchell);

George Joseph Weldrick, 12 Booterstown Avenue, Booterstown, Co. Dublin (proposed by E. T. Whittaker); and

Rev. Charles Whyte, United Free Church Manse, Towie, Glenkindie, Aberdeenshire (proposed by W. H. S. Monck).

One hundred and eighty-one presents were announced as having been received since the last meeting, including, amongst others:—

E. Anding, Kritische Untersuchungen ueber die Bewegung der Sonne, presented by the Author; Sir R. S. Ball, Popular Guide to the Heavens, third edition, presented by G. Philip & Son; J. Bosler, Théories modernes du Soleil, presented by O. Doin & fils; G. F. Dodwell, Report on the total Solar Eclipse of 1910, presented by the Author; Royal Observatory, Edinburgh, Annals, vol. iii. (Copeland and Dyson, Catalogue of Zodiacal Stars), presented by the Observatory; Groningen Astronomical Laboratory, Publications, No. 24 (Kapteyn and Weersma, Parallax Determinations), presented by the Laboratory; H. F. Newall, The Spectroscope and its Work, presented by the Author; R. A. Sampson, Tables of the four great Satellites of Jupiter, presented by Durham Observatory; G. V. Schiaparelli, Osservazioni del pianeta Marte, presented by the Schiaparelli family; Spitzbergen, Missions pour la mesure d'un arc de méridien (Mission Russe), tome i. 3, presented by the Pulkovo Observatory; L. Struve, Bearbeitung der von W. Struve in 1818–1822 angestellten Beobachtungen, presented by the Author; R. W. Willson, Laboratory Astronomy, etc., presented by the Author.

Astrographic Chart: 7 charts from Algiers Observatory, 9 charts from Bordeaux, 23 charts from Paris, and 10 charts from Toulouse, presented by the French Minister of Public Instruction; 20 charts from San Fernando Observatory and 20 charts from Tacubaya, presented by the Observatories.

Photographs of Halley's Comet: 80 lantern slides, presented by Prof. Curtis, Lick Observatory; 35 transparencies, presented by Prof. Perrine, Cordoba Observatory; 13 lantern slides by H. Knox Shaw, presented by the Helwân Observatory, and 3 transparencies presented by the Kodaikánal Observatory; 7 transparencies from photographs of nebulæ taken at Mount Wilson Observatory, presented by G. W. Ritchey.

The Grave of the Rev. James Pound.

In June 1909 attention was called* to the fact that the grave of the Rev. James Pound in Wanstead Churchyard was not suitably marked. The gravestone was broken across, the inscription on it gave no indication of his astronomical eminence, and the weather was gradually effacing the letters. The matter was mentioned at a meeting of Council of the Society, and subscriptions were invited for repairing or renewing the stone, whichever might be deemed advisable. After consultation with the Rev. Canon Corbett, the present Rector of Wanstead, who took great interest in the matter, and with an expert, it was decided to repair the old stone rather than substitute a new one, and at the same time to add a few words on a new stone indicating the historical associations. This work has now been carried out satisfactorily, and was inspected by a member of Council on November 2. Subscriptions were received towards it both at Wanstead and from Fellows; they would, however, not have been sufficient but for the timely generosity of an individual Fellow. The inscription on the old stone is as follows:—

JAMES POUND

RECTOR OF THIS PARISH
DIED NOV. 16TH 1724

ELIZABETH HIS WIFE
DIED SEPT. 10TH 1740.

ALSO LIETH THE BODY OF
WILLIAM HOPKINS OF THIS PARISH
GENTLEMAN AND CITIZEN OF LONDON
WHO DIED DEC. 24TH 1777
AGED 67 YEARS.

HERE ALSO LIETH MISS MARY HOPKINS,
DAUGHTER OF THE LATE
MR WILLIAM HOPKINS,
WHO DEPARTED THIS LIFE JULY 26TH 1780
IN HER 10TH YEAR

Below this has now been added a new stone with the following inscription:—

* *Observatory*, vol. xxxii. p. 259.

TO THE EARLY INSTRUCTION AND
FATHERLY CARE OF THE
REV. JAMES POUND, RECTOR OF WANSTEAD,
ARE DUE IN A GREAT MEASURE
THE ASTRONOMICAL DISCOVERIES,
OF HIS NEPHEW THE REV. JAMES BRADLEY:
SAVILIAN PROFESSOR OF ASTRONOMY,
OXFORD 1721–1762,
ASTRONOMER ROYAL 1742–1762.
THIS STONE WAS PLACED IN 1910 BY SOME
FELLOWS OF THE ROYAL ASTRONOMICAL SOCIETY
AND OTHERS, TO MARK AND PRESERVE HIS GRAVE.

It may perhaps be added here that an account of the Rev. James Pound was given by Mr. Walter Crouch, F.Z.S., in the *Essex Naturalist* (the journal of the Essex Field Club) for Oct.–Dec. 1893.

H. H. T.

The Systematic Motions of the Stars of Professor Boss's "Preliminary General Catalogue." By A. S. Eddington, M.A., M.Sc. (Plates 1 and 2.)

(*Communicated by the Astronomer Royal.*)

1. *Introduction.*

In the present paper I discuss the proper motions of Professor Boss's "Preliminary General Catalogue of 6188 Stars" from the standpoint of the hypothesis of two star-drifts. This catalogue supplies a want that has long been felt by those who have wished to investigate the statistics of stellar motions; and there can be little doubt that it affords data for a more complete determination of the systematic motions of the stars than has been possible with former catalogues. The stars contained are well distributed over the whole sky; they are all well-observed stars, so that the proper motions have a high degree of accuracy; all the brighter stars are included, and the fainter stars of the catalogue appear to be fairly representative, and not selected on account of the size of their proper motions. These qualities render the catalogue particularly favourable for such an investigation. Still more important is the fact that there is reason to believe a very high standard has been attained as regards elimination of systematic error; we cannot expect to obtain accordant results unless this has been successfully accomplished.

Perhaps an additional interest arises, since it is understood that Professor Boss himself is not in agreement with Professor Kapteyn's two-stream theory of stellar motions. This may have

led to an impression that the highly accurate motions of his catalogue would not show the phenomena on which that theory is based. It may be stated at once that there is no such discordance between the catalogues. Boss's proper motions show the two favoured directions of motion in the same manner that the other catalogues show them; and the hypothesis, which is based on this anomalous and unsymmetrical distribution of the motions, is in no way affected by the improved data. The issue between Professor Boss and the upholders of the two-stream theory is thus narrowed; the divergence of view does not arise from a difference in the data used, but from the method of discussion of the data. For my own part, I am convinced that the new catalogue greatly strengthens the evidence in favour of the two star-streams; for, as will be shown, it leads to results from the different parts of the sky in close accordance with one another, and thus removes some of the discrepancies shown by the less perfect data previously available.

The main purpose of the present investigation was to determine as accurately as possible the directions, velocities, and relative proportions of the two star-drifts. The work was arranged for the purpose of developing rather than demonstrating the theory. But I shall also endeavour to exhibit the strong indications of the two streams as they appeared in the course of the work. In this connection it may be well to define exactly how much is claimed for the two-drift * theory. The existence of two streams of stars appears to me beyond doubt; that is to say, there are two favoured directions of motion in which the stars appear to stream; this peculiarity is substantially the same in all parts of the sky, and it is by far the most noteworthy feature in the distribution of the stellar motions. In the light of Professor Dyson's researches on the very large proper motions, I do not see how this conclusion is to be avoided, unless we suppose that even the largest and best determined proper motions are altogether unreliable. When, however, this phenomenon is expressed as a quantitative law, and the amount of the streaming is measured, some approximation is necessary. Two forms of mathematical approximation have been used, namely, the two-drift hypothesis (which may be regarded quite apart from the tempting inference that the stars belong to two intermingled but originally separate systems), and Professor Schwarzschild's ellipsoidal hypothesis. Working on either of these theories, it has been found possible to construct an ideal representation of the universe of a fairly simple character, which will summarise quantitatively as well as qualitatively the most significant features of the arrangement of stellar motions. But such an ideal representation is essentially an approximation; it

* It is convenient to use the word *stream* for a general tendency of the stars to move in a favoured direction, without implying any hypothesis as to the cause or exact distribution of the motions; the word *drift* is used to denote a system having a distribution of individual velocities according to Maxwell's law.

must be our object, as we obtain improved data, to discover what modification may be indicated, and thus to proceed to a closer approximation. Undoubtedly a complete account of the stellar motions will need to take account of the spectral types of the stars, for there is considerable evidence of a connection between spectral type and linear motion; up to the present it has not been possible to take this heterogeneity into account. Realising, then, that the two-drift theory is to be regarded only as a first approximation, it is claimed that it yields a close approximation—perhaps as close as can be attained in the present state of our knowledge of stellar motions (see, however, § 6). Further, it correlates for us the distributions of motion observed in different parts of the sky, and shows that they are all essentially the same phenomenon, seen under different aspects.

2. *The Data of Observation.*

The material discussed in this paper consists of the directions of motion (without regard to the magnitude of the proper motion) of the stars of Boss's catalogue. It may be noted that—

(1) The proper motions given in the catalogue were corrected to reduce to Boss's value of the precession and motion of the equinox, by applying the corrections given on p. xxviii of his catalogue.

(2) All the stars of the Orion type of spectrum, so far as known, were omitted from the investigation.

(3) The fainter components of double stars, when they seemed to share the motion of the primary, the Pleiades, the stars of Boss's Taurus stream, and of the Ursa Major stream, were omitted.

With regard to (1), the application of the corrections has made little difference so far as the present research is concerned, owing to my having treated opposite portions of the sky together; but in preparing the data, I had in view ultimately a comparison of opposite regions of the sky, and for this a correct value of the precession is very important. I had previously computed all the directions of motion without applying this correction, so that the change involved a great amount of additional labour, but it incidentally provided an almost complete arithmetical check on the computation of the directions, and enabled some mistakes to be detected.

As Professor Boss's determination of the precession rests (at least formally) on a one-drift hypothesis, I was not satisfied to adopt it without examination. I know of no satisfactory analytical method of determining the precession on the two-drift theory, and all that was possible was to compare carefully the distribution of the motions, particularly the small proper motions, in opposite parts of the sphere. I made a number of diagrams of the proper motions, representing each by a dot whose cartesian co-ordinates were the components of the proper motions. By comparing the diagrams for opposite regions of the sky, with particular reference

to the points where the dots were most thickly clustered, I was able to determine with fair accuracy the most probable systematic correction required by each to bring them into agreement. Somewhat to my surprise, these diagrams seemed to confirm Professor Boss's result decidedly, at least as regards his mean correction; I accordingly adopted his values.

The peculiarities of the Orion stars are dealt with in § 5.

Local streams and "moving clusters" show themselves very distinctly in the course of the investigation, unless their direction of motion too nearly coincides with that of a drift. The case of the Ursa Major stream is rather curious. I had thought that an association of six stars would not make an appreciable difference in the distribution of the motions, and that similar associations (among fainter stars) might be quite common; accordingly these stars were not at first rejected from the work. The irregularity which they introduced was, however, sufficiently conspicuous to attract attention; the presence of a moving cluster was detected; and it was only when the names and magnitudes of the stars of this cluster were found out that it was learnt that the cluster was the well-known Ursa Major stream.

3. *Analysis of the Data.*

For the purpose of analysis the sky has been divided into thirty-four areas of approximately equal extent and of compact shape. The proper motions are regarded as projected on the tangent planes at the centres of these areas. As every area in the northern hemisphere has an exactly antipodal area in the southern hemisphere corresponding to it, and the tangent planes for each pair are parallel, we have only seventeen different planes of projection. Accordingly (following Professor Dyson) I have treated together as one region each pair of opposite areas; thus the sky is divided into seventeen regions, each region being double, and consisting of a northern and a southern portion exactly opposite to one another.

The principal object of this device was to secure a sufficient number of stars in each region without unduly extending the portion of the sky that must be treated as plane. There is the further advantage that any error in the adopted precession will affect the two portions of one region in exactly opposite directions, and its effects will thus be eliminated to a great extent. On the other hand, this treatment would lead us to overlook an effect such as that noticed by Hough and Halm,—a tendency of the drift II stars to congregate towards one half and away from the other half of the sphere. It will certainly be necessary to supplement the present work by a direct comparison of opposite parts of the sphere: this comparison must be deferred to a future paper.

The various regions are as follows—(I give the limits of the northern portions only, but it will be understood that the antipodal

part of the sky is also to be included):—Region I is the circular area Dec. $+70^\circ$ to the Pole.

Regions	II.	III.	IV.	are between
the limits of R.A.	22^h–2^h,	2^h–6^h,	6^h–10^h.	
	V.	VI.	VII.	between
the limits of R.A.	10^h–14^h,	14^h–18^h,	18^h–22^h,	respectively,

and between Dec. $+36^\circ$ and $+70^\circ$.

Regions	VIII.	IX.	X.
are between R.A.	0^h–$2^h\ 24^m$,	$2^h\ 24^m$–$4^h\ 48^m$,	$4^h\ 48^m$–$7^h\ 12^m$,
	XI.	XII.	XIII.
between R.A.	$7^h\ 12^m$–$9^h\ 36^m$,	$9^h\ 36^m$–12^h,	12^h–$14^h\ 24^m$,
	XIV.	XV.	
between R.A.	$14^h\ 24^m$–$16^h\ 48^m$,	$16^h\ 48^m$–$19^h\ 12^m$,	
	XVI.	XVII.	
between R.A.	$19^h\ 12^m$–$21^h\ 36^m$,	$21^h\ 36^m$–24^h,	

and between Dec. 0° and $+36^\circ$.

The northern parts of Regions I to VII are identical with the regions A to G respectively, into which the Groombridge stars were divided in my former investigation ("The Systematic Motions of the Stars," *Monthly Notices*, lxvii. p. 34).

The same method of treatment was employed as in the above-cited investigation. The directions of motion of all the stars of the region were calculated, and the number moving within each ten degrees of position angle counted. For the northern parts of the regions the angle θ, specifying the direction, was measured from the direction of increasing R.A. at the centre of the region, and increasing with the position angle; in fact $\theta + 90^\circ$ gives the position angle as ordinarily measured, except that a correction has been applied for the inclination of the meridian through the star to the central meridian of the region. For the southern stars the reckoning of θ was necessarily reversed in order that parallel directions in the two parts of the region might be indicated by the same value of θ.

In the seventeen tables (pp. 10–26), opposite the value of θ, is given in the second and third columns the number of stars moving in directions between $\theta - 5^\circ$ and $\theta + 5^\circ$ for the northern and southern halves of each region separately. In giving these columns I have reinserted the stars of the Taurus and Ursa Major streams, because it is of interest to note how these streams are revealed by this method of investigation; they have been excluded in all subsequent stages. The number for the double region is given in the seventh column, which has been smoothed by taking "means of three." This seventh column was analysed, that is, a combination of two single-drift distributions was found which agreed approximately with the observed distribution of the proper motions, the assumed velocities, etc. of the two drifts

being varied until it was considered that the best possible fit had been obtained. In another paper * I have given a method of finding the constants by mathematical formulæ without resorting to trial and error, but after using both methods I am convinced that trial and error gives the best results. The constants of the drifts thus found are given at the foot of the table, and the corresponding single-drift distributions, with their sum, are given in columns four, five, and six. The numbers in these columns are to the nearest third of a unit; the effect of smoothing has been allowed for in calculating them. In the last column the differences between the observed and the calculated distributions are given.

The discordances in the last column of the tables are, I think, rather larger proportionately than those which occurred in my analysis of the Groombridge proper motions. This is no doubt mainly due to the small number of stars in each region. For that reason this investigation is not very suitable for testing how exact is the agreement in this respect.

* *Monthly Notices*, lxviii. p. 588.

[Table.

REGION I. (Centre, North Pole.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	30	12	37·3	1·7	39	38·7	0
15	12	12	30·7	1·7	32·3	31	−1
25	11	16	21·3	2	23·3	19·3	−4
35	4	3	13	2	15	14·7	0
45	4	6	7·3	2·3	9·7	8	−2
55	4	3	3·7	2·7	6·3	7	+1
65	1	3	2	3	5	5·7	+1
75	3	3	1	3·3	4·3	4·7	0
85	1	3	·7	4	4·7	6·3	+2
95	7	2	·3	4·7	5	6	+1
105	5	0	·3	5·3	5·7	7·3	+2
115	6	2	·3	6	6·3	5·7	−1
125	3	1	0	6·7	6·7	6·3	0
135	4	3	0	7·3	7·3	6·7	−1
145	4	5	0	7·7	7·7	6·7	−1
155	2	2	0	7·7	7·7	7·7	0
165	7	3	0	7·7	7·7	6·7	−1
175	5	1	0	7·7	7·7	8	0
185	7	1	0	7·3	7·3	7	0
195	4	3	0	6·7	6·7	8·3	+2
205	4	6	0	6	6	7·3	+1
215	4	1	0	5·3	5·3	5·7	0
225	0	2	0	4·7	4·7	3·3	−1
235	2	1	0	4	4	3·7	0
245	3	3	·3	3·3	3·7	3·7	0
255	1	1	·3	3	3·3	4·3	+1
265	4	1	·3	2·7	3	2·7	0
275	1	0	·7	2·3	3	2·7	0
285	1	1	1	2	3	2	−1
295	2	1	2	2	4	3·3	−1
305	2	3	3·7	1·7	5·3	3·7	−2
315	1	2	7·3	1·7	9	7·7	−1
325	10	5	13	1·7	14·7	14·7	0
335	17	9	21·3	1·7	23	22	−1
345	14	11	30·7	1·7	32·3	33·7	+1
355	27	23	37·3	1·7	39	39	0
Total	217	154	236	142·7	378·7	371	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·65	0°	236
Drift II	0·45	160°	143

REGION II. (Centre R.A. 0^h, Dec. $+50°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	30	17	33·3	1	34·3	36	+2
15	13	11	33·3	1·3	34·7	34·3	0
25	15	17	28·7	1·7	30·3	24·3	−6
35	8	9	21·7	2	23·7	24·3	+1
45	10	14	14·3	2·7	17	16·7	0
55	5	4	9	3·3	12·3	14·7	+2
65	4	7	5·3	4·3	9·7	9·3	0
75	4	4	3	5·3	8·3	9·7	+1
85	4	6	1·7	6·7	8·3	8	0
95	3	3	1	7·7	8·7	7·3	−1
105	4	2	·7	8·7	9·3	7·7	−2
115	7	4	·3	9·3	9·7	9·3	0
125	4	7	·3	9·3	9·7	10	0
135	6	2	·3	8·7	9	11·3	+2
145	9	6	·3	7·7	8	10·7	+3
155	5	4	·3	6·7	7	8	+1
165	0	0	0	5·3	5·3	4	−1
175	2	1	0	4·3	4·3	2	−2
185	2	1	0	3·3	3·3	2·3	−1
195	0	1	0	2·7	2·7	1·3	−1
205	0	0	0	2	2	1·3	−1
215	1	2	0	1·7	1·7	2·3	+1
225	3	1	·3	1·3	1·7	3·3	+2
235	2	1	·3	1	1·3	2·3	+1
245	0	0	·3	1	1·3	1·3	0
255	1	0	·3	·7	1	·7	0
265	0	1	·3	·7	1	1·7	+1
275	3	0	·7	·7	1·3	2	+1
285	0	2	1	·7	1·7	2·7	+1
295	1	2	1·7	·3	2	2·3	0
305	1	1	3	·3	3·3	2·3	−1
315	1	1	5·3	·7	6	4	−2
325	6	2	9	·7	9·7	6·7	−3
335	6	4	14·3	·7	15	12·7	−2
345	11	9	21·7	·7	22·3	22·3	0
355	21	16	28·7	1	29·7	34·7	+5
Total	192	162	240·7	116	356·7	354	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·45	10°	241
Drift II	0·80	120°	116

REGION III. (Centre R.A. 4^h, Dec. $+50°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	2	3	3·7	2·3	6	6	0
15	4	4	6·3	2·7	9	12	+3
25	16	7	11	2·7	13·7	16·3	+3
35	10	8	17	3	20	21·7	+2
45	14	10	23·7	3	26·7	26·7	0
55	23	15	28	3·3	31·3	31·7	0
65	13	20	28	3·7	31·7	34	+2
75	17	14	23·7	3·7	27·3	26·7	−1
85	6	10	17	4	21	19·3	−2
95	5	6	11	4	15	12·3	−3
105	5	5	6·3	4·3	10·7	11	0
115	7	5	3·7	4·3	8	8·7	+1
125	1	3	2	4·3	6·3	5·7	−1
135	1	0	1	4·3	5·3	3·3	−2
145	3	2	·7	4·3	5	3	−2
155	2	1	·3	4·3	4·7	4	−1
165	0	4	·3	4	4·3	5	+1
175	4	4	·3	4	4·3	5·7	+1
185	4	1	0	3·7	3·7	6·7	+3
195	5	2	0	3·7	3·7	4·7	+1
205	1	1	0	3·3	3·3	3·7	0
215	2	0	0	3	3	2	−1
225	0	2	0	3	3	1·3	−2
235	0	0	0	2·7	2·7	1	−2
245	1	0	0	2·7	2·7	·7	−2
255	1	0	0	2·3	2·3	2	0
265	3	1	0	2·3	2·3	3	+1
275	3	1	0	2·3	2·3	3·3	+1
285	2	0	0	2·3	2·3	3·3	+1
295	4	0	0	2·3	2·3	2·3	0
305	0	1	·3	2	2·3	2·7	0
315	2	1	·3	2	2·3	2	0
325	1	1	·3	2·3	2·7	1·7	−1
335	0	0	·7	2·3	3	2·3	−1
345	3	2	1	2·3	3·3	3·3	0
355	4	1	2	2·3	4·3	5	+1
Total	169	135	188·7	113·7	302·3	304	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·55	60°	189
Drift II	0·20	130°	114

REGION IV. (Centre R.A. 8h, Dec. +50°.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	5	1	·3	5	5·3	5	0
15	2	2	·3	5	5·3	5·3	0
25	3	3	·7	5	5·7	4·3	−1
35	2	1	1	5·3	6·3	6	0
45	7	2	1·7	5·3	7	5·3	−2
55	2	2	2·7	5·3	8	7·7	0
65	3	7	4	5	9	8·3	−1
75	8	3	6·3	5	11·3	11·3	0
85	8	5	9·7	5	14·7	14·7	0
95	15	5	13·7	4·7	18·3	20·3	+2
105	19	9	17·3	4·3	21·7	23·3	+2
115	12	10	19·7	4	23·7	23·7	0
125	11	10	19·7	4	23·7	21·7	−2
135	11	11	17·3	3·7	21	21	0
145	7	13	13·7	3·3	17	17·7	+1
155	4	7	9·7	3·3	13	12·7	0
165	2	5	6·3	3	9·3	8·7	−1
175	6	2	4	3	7	6	−1
185	0	3	2·7	2·7	5·3	5·3	0
195	4	1	1·7	2·7	4·3	4	0
205	3	1	1	2·7	3·7	5	+1
215	3	3	·7	2·7	3·3	4·3	+1
225	1	2	·3	2·7	3	3·7	+1
235	2	0	·3	2·7	3	2·7	0
245	2	1	·3	2·7	3	3·7	+1
255	4	2	·3	2·7	3	4·3	+1
265	2	2	·3	2·7	3	4	+1
275	2	0	·3	3	3·3	3	0
285	3	0	0	3	3	2·7	0
295	1	2	0	3·3	3·3	3	0
305	2	1	0	3·3	3·3	3·3	0
315	2	2	0	3·7	3·7	3·7	0
325	4	0	·3	4	4·3	5	+1
335	4	3	·3	4	4·3	4·3	0
345	1	1	·3	4·3	4·7	4·7	0
355	2	3	·3	4·7	5	4·3	−1
Total	169	125	157·3	136·7	294	294	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·30	120°	157
Drift II	0·20	45°	137

REGION V. (Centre R.A. 12^h, Dec. $+50°$.)

θ.	Observed Distribution. North part.	Observed Distribution. South part.	Calculated Distribution. Drift I.	Calculated Distribution. Drift II.	Calculated Distribution. Sum.	Observed (smoothed).	Obs. – Calc.
5°	2	6	0	5·3	5·3	6·3	+1
15	3	3	0	5·7	5·7	5·3	0
25	0	2	0	6·3	6·3	5	−1
35	1	6	0	6·7	6·7	6	−1
45	6	3	0	7	7	8	+1
55	4	4	·3	7	7·3	8·3	+1
65	4	4	·3	7	7·3	5·7	−2
75	1	0	·3	6·7	7	4	−3
85	2	1	·7	6·3	7	3·3	−4
95	2	4	1	5·7	6·7	6	−1
105	4	5	1·7	5·3	7	7·3	0
115	5	2	2·7	4·7	7·3	9·7	+2
125	5	8	4·7	4·3	9	12·7	+4
135	11	7	7·7	3·7	11·3	16·3	+5
145	7	11	12	3·3	15·3	17·3	+2
155	9	7	16·3	3	19·3	18·3	−1
165	12	9	19	2·7	21·7	18·3	−3
175	11	7	19	2·3	21·3	21	0
185	14	10	16·3	2	18·3	20·3	+2
195	9	10	12	2	14	17	+3
205	3	5	7·7	2	9·7	10·7	+1
215	1	4	4·7	1·7	6·3	6	0
225	4	1	2·7	1·7	4·3	4·7	0
235	0	4	1·7	1·7	3·3	3·3	0
245	1	0	1	1·7	2·7	3·3	+1
255	2	3	·7	1·7	2·3	2	0
265	0	0	·3	2	2·3	3·3	+1
275	3	2	·3	2	2·3	2	0
285	0	1	·3	2	2·3	3	+1
295	1	2	0	2·3	2·3	2·3	0
305	0	3	0	2·7	2·7	3	0
315	2	1	0	3	3	2	−1
325	0	0	0	3·3	3·3	1·3	−2
335	0	1	0	3·7	3·7	1·7	−2
345	3	1	0	4·3	4·3	3·3	−1
355	10*	1	0	4·7	4·7	5·7	+1
Total	142*	138	133·3	137·3	270·7	274	...

* Including 6 stars of the Ursa Major stream.

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·50	170°	133
Drift II	0·40	55°	137

REGION VI. (Centre R.A. 16h, Dec. +50°.)

θ.	Observed Distribution. North part.	Observed Distribution. South part.	Calculated Distribution. Drift I.	Calculated Distribution. Drift II.	Calculated Distribution. Sum.	Observed (smoothed).	Obs. – Calc.
5°	2	2	1	2·3	3·3	3	0
15	2	2	1	3	4	3·3	−1
25	1	1	1	3·7	4·7	4	−1
35	4	2	1	4·7	5·7	4	−2
45	2	2	·7	6	6·7	6·7	0
55	7	3	1	7	8	8·7	+1
65	10	2	1	8	9	10·3	+1
75	4	5	1	8·3	9·3	9·7	0
85	2	6	1	8·3	9·3	7·7	−2
95	2	4	1·3	8	9·3	6·7	−3
105	6	0	1·3	7	8·3	8	0
115	5	7	1·7	6	7·7	7·3	0
125	1	3	2	4·7	6·7	6	−1
135	1	1	2·7	3·7	6·3	4·3	−2
145	6	1	3·7	3	6·7	4·7	−2
155	1	4	4·7	2·3	7	8	+1
165	6	6	6·3	1·7	8	8·7	+1
175	3	6	8	1·3	9·3	10·3	+1
185	3	7	10·3	1	11·3	10·7	−1
195	3	10	12·7	1	13·7	12	−2
205	7	6	15	·7	15·7	15	−1
215	11	8	16·7	·7	17·3	14·7	−3
225	8	4	17	·7	17·7	18	0
235	8	15	16·7	·7	17·3	16·3	−1
245	7	7	15	·3	15·3	16·7	+1
255	8	5	12·7	·3	13	13·7	+1
265	4	10	10·3	·3	10·7	12·3	+2
275	3	7	8	·3	8·3	9·7	+1
285	1	4	6·3	·7	7	6·3	−1
295	3	1	4·7	·7	5·3	5	0
305	3	3	3·7	·7	4·3	5	+1
315	3	2	2·7	·7	3·3	5	+2
325	2	2	2	1	3	4·3	+1
335	1	3	1·7	1	2·7	3·3	+1
345	1	1	1·3	1·3	2·7	2·3	0
355	1	0	1·3	1·7	3	2·3	−1
Total	142	152	198·3	102·7	301	294	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	0·85	225°	198
Drift II	0·80	80°	103

REGION VII. (Centre R.A. 20^h, Dec. $+50°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	5	12	11	5·3	16·3	18·3	+2
15	8	9	7·7	7	14·7	17	+2
25	7	10	5·3	8·7	14	15·3	+1
35	5	7	3·7	10·7	14·3	14	0
45	5	8	2·7	13	15·7	13·7	−2
55	7	9	2	15	17	17	0
65	9	13	1·3	16·3	17·7	19·3	+2
75	10	10	1	16·7	17·7	18·7	+1
85	9	5	1	16·3	17·3	16	−1
95	10	4	·7	15	15·7	13·7	−2
105	10	3	·7	13	13·7	11	−3
115	2	4	·7	10·7	11·3	10	−1
125	5	6	·7	8·7	9·3	10·3	+1
135	8	6	·7	7	7·7	10	+2
145	2	3	·7	5·3	6	7·7	+2
155	3	1	·7	4·3	5	5	0
165	3	3	·7	3·3	4	3·7	0
175	1	0	·7	2·7	3·3	3·3	0
185	0	3	·7	2·3	3	2	−1
195	1	1	1	2	3	2·7	0
205	2	1	1	1·7	2·7	2	−1
215	0	1	1·3	1·3	2·7	2·7	0
225	1	3	2	1·3	3·3	3	0
235	2	2	2·7	1·3	4	4	0
245	1	3	3·7	1	4·7	3·7	−1
255	2	1	5·3	1	6·3	6·3	0
265	10	2	7·7	1	8·7	9	0
275	8	4	11	1·3	12·3	14	+2
285	8	10	15	1·3	16·3	16·3	0
295	9	10	19·3	1·3	20·7	19·3	−1
305	14	7	23	1·7	24·7	24·3	0
315	14	19	25	2	27	26·3	−1
325	16	9	25	2·3	27·3	26·3	−1
335	8	13	23	2·7	25·7	22	−4
345	12	8	19·3	3·3	22·7	20·7	−2
355	10	11	15	4·3	19·3	19·3	0
Total	227	221	242·7	212·3	455	448	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·05	320°	243
Drift II	0·75	75°	212

REGION VIII. (Centre R.A. $1^h\ 12^m$, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	10	20	21·7	1·3	23	26	+3
15	11	17	25·3	1·7	27	28	+1
25	13	13	25·3	2	27·3	25·3	−2
35	11	11	21·7	2·3	24	23·7	0
45	8	15	16·3	3	19·3	20·7	+1
55	8	9	11	3·7	14·7	16	+1
65	4	4	6·7	4·7	11·3	11·7	0
75	5	5	4	5·7	9·7	9	−1
85	7	2	2·3	6·7	9	8·7	0
95	3	4	1·3	7·7	9	8·3	−1
105	5	4	·7	8·7	9·3	8·3	−1
115	6	3	·7	9	9·7	8·7	−1
125	5	3	·3	9	9·3	8·7	−1
135	7	2	·3	8·7	9	9	0
145	5	5	·3	7·7	8	10·7	+3
155	8	5	·3	6·7	7	9·3	+2
165	2	3	0	5·7	5·7	7·3	+2
175	2	2	0	4·7	4·7	3·7	−1
185	0	2	0	3·7	3·7	2·3	−1
195	0	1	0	3	3	2	−1
205	3	0	0	2·3	2·3	1·7	−1
215	1	0	0	2	2	1·3	−1
225	0	0	0	1·7	1·7	·7	−1
235	1	0	0	1·3	1·3	1	0
245	1	1	·3	1	1·3	1·3	0
255	0	1	·3	1	1·3	2	+1
265	1	2	·3	1	1·3	2	+1
275	2	0	·3	·7	1	2·3	+1
285	0	2	·7	·7	1·3	2·7	+1
295	2	2	·7	·7	1·3	3·3	+2
305	1	3	1·3	·7	2	3·7	+2
315	2	1	2·3	·7	3	3·3	0
325	2	1	4	·7	4·7	2·3	−2
335	0	1	6·7	1	7·7	4	−4
345	3	5	11	1	12	9·7	−2
355	10	10	16·3	1	17·3	19·3	+2
Total	149	159	182·7	122·7	305·3	308	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·45	20°	183
Drift II	0·70	120°	123

REGION IX. (Centre R.A. $3^h\ 36^m$, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	13*	3	9·3	1·3	10·7	10	−1
15	27*	12	14·7	1·7	16·3	13·7	−3
25	11*	10	20·3	2	22·3	22	0
35	16	17	24	2·7	26·7	25·3	−1
45	16	11	24	3·3	27·3	28·3	+1
55	16	9	20·3	4·3	24·7	25	0
65	12	11	14·7	5·7	20·3	22	+2
75	8	10	9·3	7	16·3	17·7	+1
85	6	6	5·3	8·3	13·7	13·3	0
95	6	4	3	9·3	12·3	10·7	−2
105	8	2	1·7	9·7	11·3	10	−1
115	4	6	1	9·7	10·7	9·3	−1
125	2	6	·7	9·3	10	8	−2
135	3	3	·3	8·3	8·7	7·3	−1
145	3	5	·3	7	7·3	8·7	+1
155	5	7	·3	5·7	6	6·7	+1
165	0	0	0	4·3	4·3	4·7	0
175	1	1	0	3·3	3·3	3·3	0
185	3	5	0	2·7	2·7	3·7	+1
195	1	0	0	2	2	3·3	+1
205	0	1	0	1·7	1·7	·7	−1
215	0	0	0	1·3	1·3	2	+1
225	1	4	0	1	1	1·7	+1
235	0	0	0	1	1	2	+1
245	1	0	0	·7	·7	·7	0
255	1	0	0	·7	·7	·7	0
265	0	0	0	·7	·7	·3	0
275	0	0	0	·7	·7	0	−1
285	0	0	·3	·3	·7	0	−1
295	0	0	·3	·3	·7	·3	0
305	1	0	·3	·7	1	·7	0
315	0	1	·7	·7	1·3	1	0
325	1	0	1	·7	1·7	1	−1
335	0	1	1·7	·7	2·3	2	0
345	1	3	3	1	4	3·3	−1
355	6*	2	5·3	1	6·3	5·7	−1
Total	173*	140	162	120·7	282·7	275	...

* Including the stars (38 in number) belonging to the Taurus stream.

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·55	40°	162
Drift II	0·80	110°	121

REGION X. (Centre R.A. 6^h, Dec. $+17°$.)

θ	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	4	2	4	1·3	5·3	4·7	−1
15	1	2	6	1·7	7·7	5·7	−2
25	3	5	8·3	2·3	10·7	6·3	−4
35	2	6	11	2·7	13·7	11	−3
45	8	9	14·3	3·7	18	17·3	−1
55	12	15	17·7	4·7	22·3	23·7	+1
65	16	11	20	5·7	25·7	32·3	+7
75	23	20	21	7	28	32·3	+4
85	11	16	20	8·3	28·3	32·3	+4
95	13	14	17·7	9·7	27·3	24	−3
105	13	5	14·3	10·3	24·7	21·3	−3
115	10	9	11	10·7	21·7	17·7	−4
125	9	7	8·3	10·3	18·7	19·3	+1
135	7	16	6	9·7	15·7	18·3	+3
145	9	7	4	8·3	12·3	15·3	+3
155	3	4	3	7	10	11·3	+1
165	6	5	2	5·7	7·7	8·7	+1
175	1	7	1·7	4·7	6·3	8·3	+2
185	2	4	1·3	3·7	5	6·3	+1
195	3	2	1	2·7	3·7	5	+1
205	1	3	·7	2·3	3	3·3	0
215	1	0	·7	1·7	2·3	3	+1
225	1	3	·7	1·3	2	2	0
235	1	0	·7	1·3	2	2	0
245	0	1	·7	1	1·7	1·7	0
255	2	1	·3	1	1·3	1·3	0
265	0	0	·7	·7	1·3	1·3	0
275	1	0	·7	·7	1·3	·7	−1
285	0	1	·7	·7	1·3	1·3	0
295	2	0	·7	·7	1·3	1·3	0
305	1	0	·7	·7	1·3	1	0
315	0	0	1	·7	1·7	·7	−1
325	1	0	1·3	·7	2	1·7	0
335	1	3	1·7	1	2·7	3·3	+1
345	3	2	2	1	3	4·7	+2
355	3	2	3	1·3	4·3	5·3	+1
Total	174	182	208·7	136·7	345·3	356	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·00	75°	209
Drift II	0·75	115°	137

REGION XI. (Centre R.A. $8^h\ 24^m$, Dec. $+17°$.)

θ.	Observed Distribution. North part.	Observed Distribution. South part.	Calculated Distribution. Drift I.	Calculated Distribution. Drift II.	Calculated Distribution. Sum.	Observed (smoothed).	Obs. – Calc.
5°	2	1	·7	2·7	3·3	3·3	0
15	3	4	1	3·3	4·3	3·7	−1
25	0	1	1·3	4	5·3	4·7	−1
35	3	3	1·7	4·7	6·3	5	−1
45	2	6	2	5·7	7·7	6·7	−1
55	1	5	2·7	6·3	9	9·7	+1
65	7	8	4	6·7	10·7	10·7	0
75	6	5	5·7	7	12·7	11·7	−1
85	7	2	8	7	15	14	−1
95	9	13	11·7	6·7	18·3	20·3	+2
105	18	12	15·7	6·3	22	26·3	+4
115	16	11	20·3	5·7	26	29·7	+4
125	21	11	24	4·7	28·7	30·7	+2
135	17	16	26·3	4	30·3	31	+1
145	14	14	26·3	3·3	29·7	28·3	−1
155	14	10	24	2·7	26·7	24	−3
165	6	14	20·3	2·3	22·7	21·7	−1
175	9	12	15·7	2	17·7	17·7	0
185	3	9	11·7	1·7	13·3	14	+1
195	2	7	8	1·3	9·3	9·3	0
205	2	5	5·7	1·3	7	7·7	+1
215	3	4	4	1	5	5·7	+1
225	1	2	2·7	1	3·7	4	0
235	2	0	2	1	3	2	−1
245	0	1	1·7	1	2·7	2	−1
255	1	2	1·3	·7	2	2	0
265	1	1	1	·7	1·7	1·7	0
275	0	0	·7	1	1·7	·7	−1
285	0	0	·7	1	1·7	·3	−1
295	0	1	·7	1	1·7	1·3	0
305	0	3	·7	1	1·7	2	0
315	1	1	·3	1·3	1·7	2·7	+1
325	1	2	·3	1·3	1·7	2·3	+1
335	1	1	·7	1·7	2·3	3·3	+1
345	4	1	·7	2	2·7	2·3	0
355	0	0	·7	2·3	3	2·7	0
Total	177	188	254·7	107·3	362	365	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·05	140°	255
Drift II	0·60	80°	107

REGION XII. (Centre R.A. $10^h\ 48^m$, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	5	4	·3	5·7	6	7	+1
15	3	5	·3	7	7·3	8·3	+1
25	4	4	·3	8·3	8·7	8	−1
35	6	2	·3	9·7	10	9·7	0
45	5	8	·3	10·3	10·7	12	+1
55	2	13	·7	10·7	11·3	11	0
65	1	4	·7	10·3	11	10	−1
75	7	3	1	9·7	10·7	8·3	−2
85	7	3	1·7	8·3	10	10	0
95	5	5	2·7	7	9·7	9·3	0
105	4	4	4·3	5·7	10	9·7	0
115	3	8	7	4·7	11·7	11	−1
125	5	9	10·7	3·7	14·3	13·7	−1
135	8	8	15·7	2·7	18·3	18·3	0
145	15	10	21	2·3	23·3	23·3	0
155	16	13	25·3	1·7	27	27·7	+1
165	20	9	27	1·3	28·3	27·7	−1
175	5	20	25·3	1·3	26·7	25·7	−1
185	8	15	21	1	22	22·7	+1
195	7	13	15·7	1	16·7	17	0
205	3	5	10·7	·7	11·3	12·3	+1
215	3	6	7	·7	7·7	7·3	0
225	1	4	4·3	·7	5	4·7	0
235	0	0	2·7	·7	3·3	2·3	−1
245	1	1	1·7	·7	2·3	2	0
255	1	3	1	·7	1·7	3	+1
265	0	3	·7	·7	1·3	3·7	+2
275	1	3	·7	1	1·7	3·3	+2
285	0	3	·3	1	1·3	2·3	+1
295	0	0	·3	1·3	1·7	1	−1
305	0	0	·3	1·3	1·7	0	−2
315	0	0	·3	1·7	2	·3	−2
325	0	1	·3	2·3	2·7	·7	−2
335	0	1	·3	2·7	3	1·3	−2
345	0	2	·3	3·7	4	2·3	−2
355	1	3	·3	4·7	5	5	0
Total	147	195	212·7	136·7	349·3	342	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·20	165°	213
Drift II	0·75	55°	137

REGION XIII. (Centre R.A. $13^h\ 12^m$, Dec. $+17°$.)

θ.	Observed Distribution. North part.	Observed Distribution. South part.	Calculated Distribution. Drift I.	Calculated Distribution. Drift II.	Calculated Distribution. Sum.	Observed (smoothed).	Obs. – Calc.
5°	0	1	·3	3·3	3·7	3	−1
15	3	4	·3	4·3	4·7	5·3	+1
25	3	5	·3	5·3	5·7	8	+2
35	5	4	·3	6·7	7	8·7	+2
45	4	5	·3	7·7	8	8·3	0
55	5	2	·3	8·3	8·7	6·7	−2
65	2	2	·7	8·3	9	6·3	−3
75	3	5	·7	7·7	8·3	6·3	−2
85	3	4	1	6·7	7·7	8·3	+1
95	4	6	1·7	5·3	7	6·7	0
105	2	1	2·7	4·3	7	7	0
115	8	0	4·3	3·3	7·7	7	−1
125	6	4	6·3	2·3	8·7	10	+1
135	6	6	9·7	1·7	11·3	14·3	+3
145	12	9	13·3	1·3	14·7	14·7	0
155	2	9	16·7	1	17·7	17·3	0
165	12	8	18·7	·7	19·3	13·3	−6
175	5	4	18·7	·7	19·3	17·3	−2
185	9	14	16·7	·7	17·3	16·7	−1
195	8	10	13·3	·7	14	15·3	+1
205	3	2	9·7	·3	10	9	−1
215	2	2	6·3	·3	6·7	5·3	−1
225	4	3	4·3	·3	4·7	4	−1
235	0	1	2·7	·3	3	4	+1
245	2	2	1·7	·3	2	2	0
255	0	1	1	·3	1·3	1·7	0
265	0	0	·7	·3	1	·7	0
275	0	1	·7	·3	1	·7	0
285	0	1	·3	·7	1	2	+1
295	2	2	·3	·7	1	1·7	+1
305	0	0	·3	·7	1	1·3	0
315	0	0	·3	·7	1	0	−1
325	0	0	·3	1	1·3	·3	−1
335	0	1	·3	1·3	1·7	1	−1
345	1	1	0	1·7	1·7	1·3	0
355	0	1	0	2·3	2·3	1·3	−1
Total	116	121	155·3	92	247·3	237	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·25	170°	155
Drift II	0·90	60°	92

REGION XIV. (Centre R.A. $15^h\ 36^m$, Dec. $+17°$.)

θ.	Observed Distribution. North part.	Observed Distribution. South part.	Calculated Distribution. Drift I.	Calculated Distribution. Drift II.	Calculated Distribution. Sum.	Observed (smoothed).	Obs. – Calc.
5°	3	1	·7	2·3	3	3	0
15	1	2	·7	3·3	4	3·7	0
25	3	1	1	4·3	5·3	5·3	0
35	6	3	1	5·3	6·3	7·3	+1
45	6	3	1	6·7	7·7	9·7	+2
55	6	5	1	7·7	8·7	10·3	+2
65	4	7	1·3	8·7	10	10	0
75	3	5	1·7	9	10·7	10	−1
85	7	4	2	8·7	10·7	8·7	−2
95	4	3	2·3	7·7	10	8	−2
105	5	1	3	6·7	9·7	9	−1
115	9	5	4	5·3	9·3	11·7	+2
125	8	7	5	4·3	9·3	12·7	+3
135	8	1	6·3	3·3	9·7	12·7	+3
145	8	6	7·7	2·3	10	11·3	+1
155	8	3	9·3	2	11·3	12·3	+1
165	8	4	10·7	1·3	12	11	−1
175	2	8	11·3	1	12·3	10	−2
185	3	5	11·7	1	12·7	9·3	−3
195	5	5	11·3	·7	12	10	−2
205	4	8	10·7	·7	11·3	12·7	+1
215	9	7	9·3	·7	10	12·7	+3
225	2	8	7·7	·3	8	10·7	+3
235	3	3	6·3	·3	6·7	7·3	+1
245	0	6	5	·3	5·3	4·7	−1
255	1	1	4	·3	4·3	3·3	−1
265	0	2	3	·3	3·3	3	0
275	2	3	2·3	·3	2·7	2·7	0
285	0	1	2	·3	2·3	2	0
295	0	0	1·7	·7	2·3	1·3	−1
305	1	2	1·3	·7	2	1·3	−1
315	0	1	1	·7	1·7	1·3	0
325	0	0	1	1	2	1	−1
335	2	0	1	1	2	2·3	0
345	3	2	1	1·3	2·3	3	+1
355	1	1	·7	2	2·7	3·7	+1
Total	135	124	151	102·7	253·7	259	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	0·75	185°	151
Drift II	0·85	75°	103

REGION XV. (Centre R.A. 18^h, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	2	3	4	1·7	5·7	5·7	0
15	5	0	4	2·3	6·3	6	0
25	7	1	3·7	3	6·7	5·3	−1
35	2	1	3·7	4	7·7	8	0
45	2	11	3·7	5·3	9	8·7	0
55	5	5	3·7	7	10·7	11·7	+1
65	5	7	3·3	8·7	12	11	−1
75	8	3	3·3	10	13·3	13·3	0
85	13	4	3·3	11	14·3	13·3	−1
95	6	6	3·7	11	14·7	15	0
105	9	7	3·7	10	13·7	13·3	0
115	3	9	3·7	8·7	12·3	13·3	+1
125	4	8	3·7	7	10·7	11	0
135	3	6	4	5·3	9·3	9·7	0
145	2	6	4	4	8	7·3	−1
155	3	2	4·3	3	7·3	7	0
165	3	5	4·3	2·3	6·7	6·3	0
175	2	4	4·7	1·7	6·3	7	+1
185	3	4	5	1	6	5	−1
195	1	1	5·3	1	6·3	5	−1
205	1	5	5·3	·7	6	6	0
215	5	5	5·3	·7	6	6·7	+1
225	0	4	5·7	·7	6·3	6·3	0
235	3	2	5·7	·3	6	4·3	−2
245	0	4	6	·3	6·3	5·3	−1
255	5	2	6	·3	6·3	6·7	0
265	0	9	6	·3	6·3	8·7	+2
275	2	8	5·7	·3	6	9	+3
285	4	4	5·7	·3	6	8	+2
295	3	3	5·3	·3	5·7	5·7	0
305	2	1	5·3	·3	5·7	5·3	0
315	4	3	5·3	·7	6	5	−1
325	4	1	5	·7	5·7	5·3	0
335	2	2	4·7	·7	5·3	6	+1
345	5	4	4·3	1	5·3	6·7	+1
355	3	4	4·3	1	5·3	7	+2
Total	131	154	164·7	116·7	281·3	285	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	0·15	255°	165
Drift II	0·95	90°	117

REGION XVI. (Centre R.A. $20^h\ 24^m$, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	5	9	17·3	1·3	18·7	14·3	−4
15	3	7	15·3	1·7	17	14·3	−3
25	6	13	12·3	2·3	14·7	13	−2
35	6	4	9·7	3·3	13	17	+4
45	11	11	7	4·3	11·3	14	+3
55	3	7	5	5·7	10·7	13·3	+3
65	6	2	3·7	7·3	11	9·7	−1
75	6	5	2·7	9	11·7	11	−1
85	6	8	2	10·3	12·3	11·3	−1
95	4	5	1·3	11·3	12·7	10·7	−2
105	6	3	1	11·3	12·3	10·3	−2
115	8	5	1	10·3	11·3	12	+1
125	8	6	·7	9	9·7	12·3	+3
135	6	4	·7	7·3	8	10·3	+2
145	4	3	·7	5·7	6·3	6·3	0
155	1	1	·7	4·3	5	4	−1
165	0	3	·3	3·3	3·7	1·7	−2
175	0	0	·3	2·3	2·7	2	−1
185	1	2	·3	1·7	2	2	0
195	2	1	·7	1·3	2	2·7	+1
205	0	2	·7	1	1·7	2·7	+1
215	1	2	·7	1	1·7	1·7	0
225	0	0	·7	·7	1·3	1·7	0
235	0	2	1	·7	1·7	1·7	0
245	2	1	1	·7	1·7	2	0
255	1	0	1·3	·7	2	1·7	0
265	1	0	2	·3	2·3	1·3	−1
275	1	1	2·7	·3	3	3	0
285	3	3	3·7	·3	4	6	+2
295	2	8	5	·3	5·3	7	+2
305	5	0	7	·7	7·7	8·3	+1
315	5	5	9·7	·7	10·3	11·3	+1
325	10	9	12·3	·7	13	14·3	+1
335	4	10	15·3	·7	16	18·7	+3
345	10	13	17·3	1	18·3	18·7	0
355	10	9	18·3	1	19·3	18·7	−1
Total	147	164	181·3	124	305·3	311	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·00	355°	181
Drift II	0·90	100°	124

REGION XVII. (Centre R.A. $22^h\ 48^m$, Dec. $+17°$.)

θ.	Observed Distribution.		Calculated Distribution.			Observed (smoothed).	Obs. – Calc.
	North part.	South part.	Drift I.	Drift II.	Sum.		
5°	7	12	20	1·3	21·3	19·7	−2
15	8	7	18·7	1·7	20·3	14·3	−6
25	5	4	14·7	2	16·7	15	−2
35	15	6	10·7	2·7	13·3	13·7	0
45	6	5	6·7	3·3	10	13·3	+3
55	6	2	4	4·3	8·3	9	+1
65	5	3	2·3	5·7	8	9	+1
75	4	7	1·3	6·7	8	7·3	−1
85	1	2	·7	7·7	8·3	7	−1
95	4	3	·7	8·7	9·3	5·7	−4
105	5	2	·3	8·7	9	8	−1
115	4	6	·3	8·7	9	8·7	0
125	4	5	·3	7·7	8	9	+1
135	5	3	0	6·7	6·7	8·7	+2
145	7	2	0	5·7	5·7	7	+1
155	4	0	0	4·3	4·3	5·3	+1
165	1	2	0	3·3	3·3	2·7	−1
175	1	0	0	2·7	2·7	1·3	−1
185	0	0	0	2	2	·3	−2
195	0	0	0	1·7	1·7	·3	−1
205	1	0	0	1·3	1·3	·7	−1
215	0	1	0	1	1	1	0
225	0	1	0	1	1	1·7	+1
235	1	2	0	·7	·7	2	+1
245	0	2	·3	·7	1	1·7	+1
255	0	0	·3	·7	1	1	0
265	0	1	·3	·7	1	1·7	+1
275	1	3	·7	·7	1·3	2	+1
285	1	0	·7	·3	1	2·7	+2
295	0	3	1·3	·7	2	1·7	0
305	0	1	2·3	·7	3	1·7	−1
315	0	1	4	·7	4·7	3	−2
325	4	3	6·7	·7	7·3	7	0
335	5	8	10·7	·7	11·3	12·7	+1
345	8	10	14·7	1	15·7	18·7	+3
355	9	16	18·7	1	19·7	20·7	+1
Total	122	123	141·3	107·7	249	245	...

Result of analysis—

	Velocity.	Direction.	Number of Stars.
Drift I	1·45	5°	141
Drift II	0·80	105°	108

Leaving aside the question of the precise law of distribution, I will point out the direct evidence for the existence of two streams. This evidence is mainly comprised in figs. 1–9, which show diagrammatically the data for nine out of the seventeen regions.

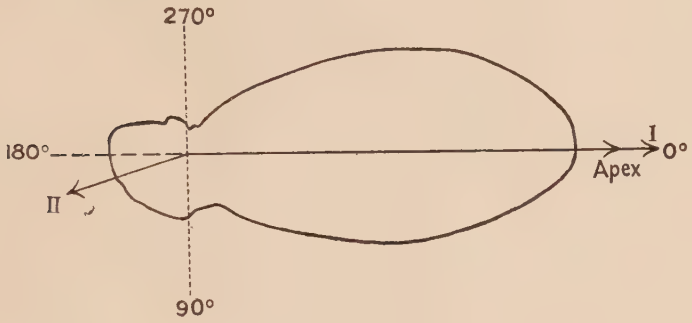


Fig. 1.—Region I; Centre, North Pole.

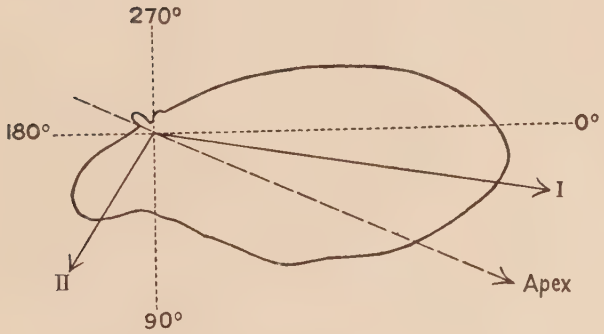


Fig. 2.—Region II; Centre R.A. 0^h, Dec. $+50°$.

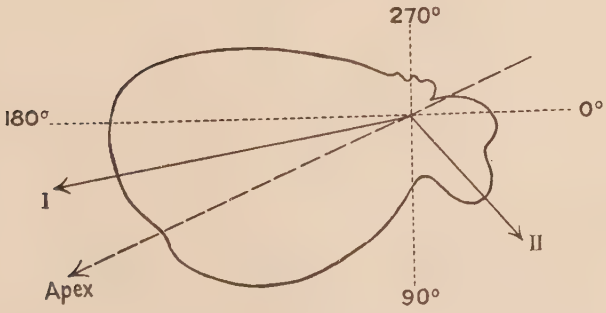


Fig. 3.—Region V; Centre R.A. 12^h, Dec. $+50°$.

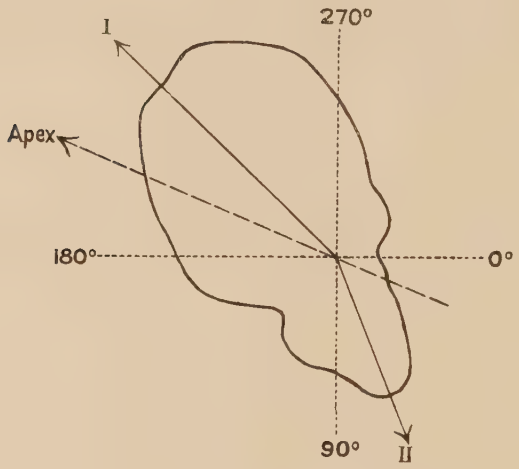


FIG. 4.—Region VI; Centre R.A. 16^h, Dec. $+50°$.

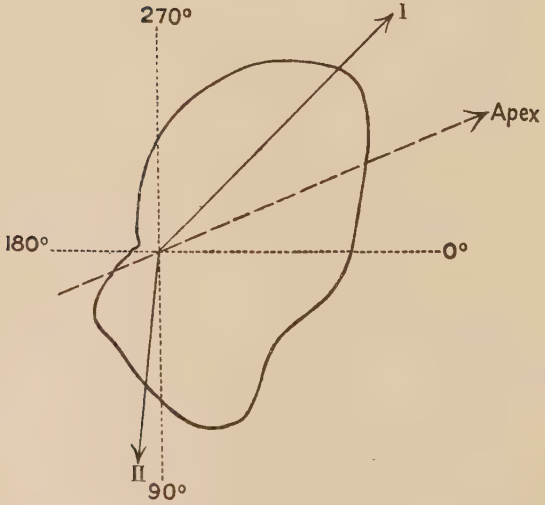


FIG. 5.—Region VII; Centre R.A. 20^h, Dec. $+50°$.

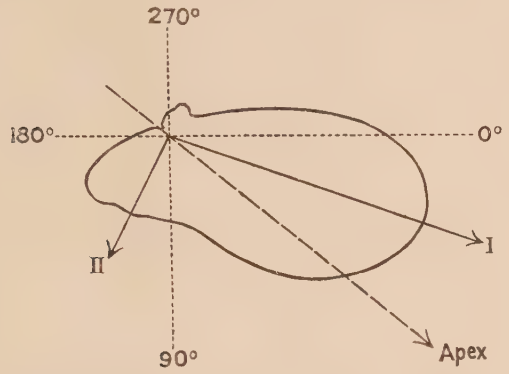


Fig. 6.—Region VIII; Centre R.A. $1^h\ 12^m$, Dec. $+17°$.

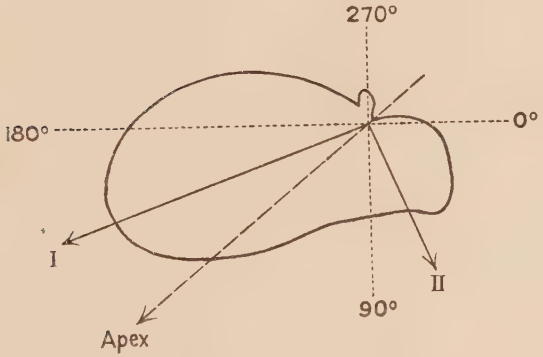


Fig. 7.—Region XII; Centre R.A. $10^h\ 48^m$, Dec. $+17°$.

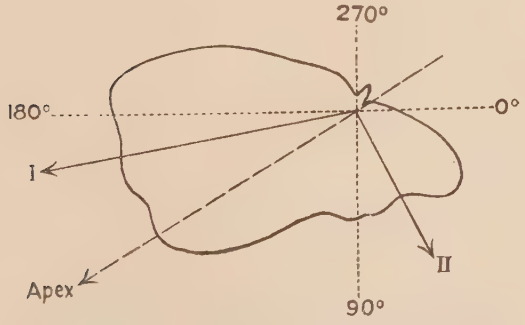


Fig. 8.—Region XIII; Centre R.A. $13^h\ 12^m$, Dec. $+17°$.

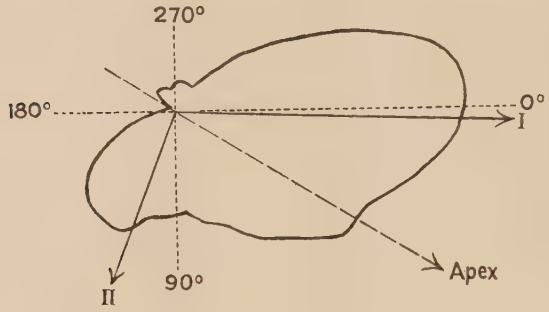


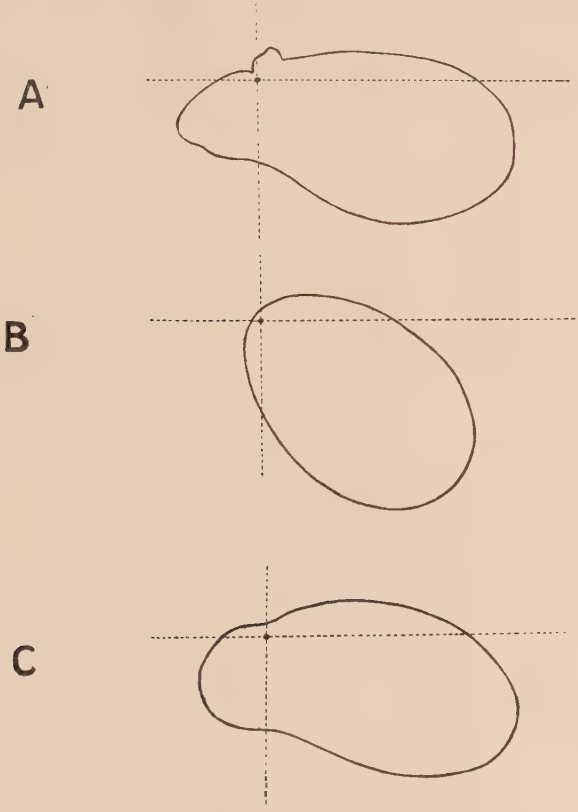
FIG. 9.—Region XVII ; Centre R.A. $22^h\ 48^m$, Dec. $+17°$.

The curves are drawn so that the distance from the origin to the curve in any direction is proportional to the number of stars moving in that direction.* I have chosen for reproduction the nine regions where the aspect of the drifts is such that the phenomenon we are looking for *ought* to be most clearly shown. It will be understood that there are parts of the sky where, on theoretical grounds, we should not expect to see the two drifts clearly differentiated. The fact that in such regions the existence of the two drifts is not very marked is, of course, evidence in support of the theory, which is, in its way, as important as the fact that in other parts of the sky the drifts are shown plainly separated. But those who wish to examine this negative evidence will probably be satisfied to refer to the tables and analysis. Of the eight regions which are not reproduced in these plates, two are reproduced and considered in detail later ; regions XIV and XVI are referred to in § 6, and in the remaining four regions the existence of two drifts would probably not be recognised in the diagrams by the casual reader, although I find no difficulty in separating them in the tables.

Turning, then, to any one of the nine diagrams, the arrow marked Apex points towards the antapex of the solar motion determined by Professor Boss from these proper motions. If the individual motions of the stars were haphazard, there ought to be approximate symmetry about this line, the greatest number of stars moving along it, and the fewest in the opposite direction. Every one of the figures, however, shows an extraordinary asymmetry, and the assumption of random motions is very far from being confirmed. I have marked on the figures, by continuous arrows, the directions towards the apices of the two drifts, not the directions determined from the region itself, but from the results of them all collectively (see p. 35). It will be seen that a streaming of the stars in those two directions is always strongly

* For constructing the curves I have smoothed the numbers in the tables by taking "means of three" a second time ; I find that these polar diagrams are much clearer when freed from trivial irregularities.

indicated. Fig. 10 will perhaps make the argument plainer. The curve A is the observed curve for region VIII. The curve B is the nearest approximation to it which I can obtain on the hypothesis of random distribution, together with the solar motion (assumed in the direction determined by Professor Boss). The



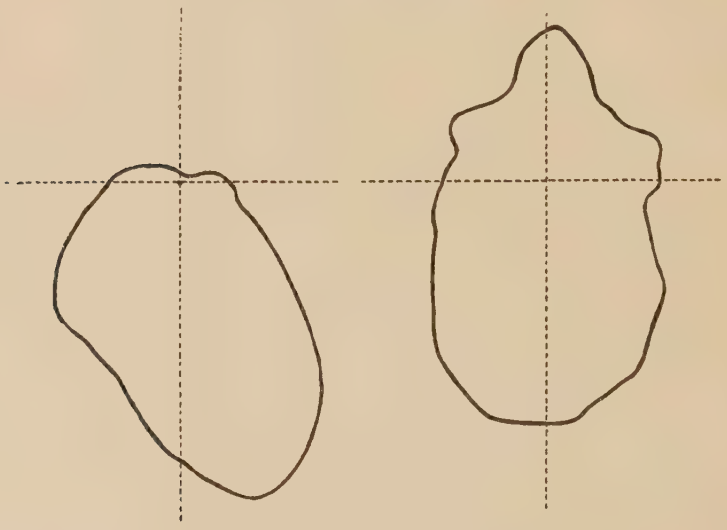
A, observed distribution ; B, single-drift approximation ; C, double-drift approximation.

Fig. 10.—Observed and theoretical distributions for Region VIII.

curve C is the nearest approximation by assuming two drifts (in directions given by the final results of this paper).

Without insisting that the differences between A and C are purely accidental, it will surely be admitted that, whereas the curve B has hardly any resemblance to the observed curve, the curve C represents all the main features of the distribution, and from it we can, if we choose, proceed to investigate the irregularities of detail.

The other eight regions can be represented as the result of two drifts in a precisely analogous manner, as they are clearly curves of a similar type. However, asymmetry and a bilobed appearance are not the only ways in which the presence of two streams can be indicated. In fig. 11 are compared curves for regions X and XV, constructed similarly to those previously given. Region X is a unique region, in that it contains nearly centrally the point called by Kapteyn the *true vertex*. At this point of the sky the two drifts have precisely the same transverse motion; they could only be distinguished in the radial motions of the stars, with which we are not here concerned. It follows that at this point the



Region X.
(Approximately a single-drift curve.)

Region XV.
(Double-drift curve.)

FIG. 11.

distribution of the proper motions is a single-drift distribution (really due to two coincident drifts), and the same must be approximately true for region X. Actually the distance between the centre of region X and the true vertex is not altogether negligible, and there is some indication of two drifts. But we may take the curve as roughly the type of curve which would result in every part of the sky if the stars formed one drift only—*i.e.* a random distribution plus the solar motion. It agrees in a general way with the theoretical single-drift curve, and is interesting as showing the kind and amount of roughness we may expect in the practical realisation of the theory. The curve for region XV, on the other hand, is very nearly symmetrical, and at first sight shows no signs of two drifts; but when it is compared with region X we notice

that, though the lower halves of the curves are similar, the upper part, instead of closely circling the origin, ceases to approach it, and makes a wide sweep above it. There is a disturbing factor in this second case, namely, the presence of another drift. The elongation of the lower half of the curve is due to drift II; it is the presence of the drift I (which in this particular region has practically no transverse velocity, and consists of stars moving with almost equal frequency in all directions) which prevents the curve from being completed in the same manner as the left-hand curve. Thus, although neither region shows, nor is expected to show, asymmetry, there is a distinction between them, exactly in accordance with anticipation.

4. *Comparison of the Results from the Seventeen Regions, and determination of the Apices of the Drifts.*

We may now collect and compare the results of the analysis of the seventeen regions, which are given below the respective tables, and determine from them the amount and direction of each drift motion. There are two ways of proceeding with this determination, which are analogous respectively to Bessel's and Airy's methods of determining the solar motion.

By the first method, since we know for seventeen points of the sphere the direction in the sky in which a drift is moving, we can draw seventeen great circles representing these directions, and examine to what point of the celestial sphere these converge most nearly. In the present case this method, although instructive, is not so accurate as Airy's analytical method, because it neglects half—perhaps the most valuable half—of the material; the speeds of the drift motions (the quantities hV in the mathematical theory) are not taken into account. These speeds are given by the analysis independently of the distances of the stars, so that Airy's method is without the disadvantages which attend its use in the solar-motion problem. For this reason all the definitive results have been derived by Airy's method; but Bessel's method has been taken advantage of in order to exhibit graphically the convergence of the directions of the drift motions from the various parts of the sky.

Figs. 12 and 13 (Plates 1, 2), for drifts I and II respectively, show this convergence. Imagine the great circles traced on the sky, and a photograph taken of the part of the sky where they converge.* The great circles will appear on such a photograph as straight lines. These are shown in the two diagrams: the Roman numeral attached to each great circle indicates the region from which it proceeds. Each diagram represents an area of the sky measuring about 60° by 30°; a good idea of this area can be gained from the comparison,—the diagram is to the whole sky as a map of

* I have here followed the suggestion of Professor Turner, *Monthly Notices*, lxx. p. 204. The "plate-centres" used are R.A. 6^h Dec. 0°, and the north pole, respectively.

Northern Africa from the Congo to the Mediterranean is to the terrestrial globe. On each diagram I have marked the definitive position of the apex of the corresponding drift, determined by Airy's method. The convergence of the directions for drift I is so striking as to need no comment. (It may be worth while to remark that the solar antapex determined by Boss is just outside the diagram, so that the drift I motion is quite clearly distinct from the solar motion.) Owing to the smaller velocity of drift II it is not possible to determine its direction in any region with the same degree of accuracy, and consequently there is much greater deviation in the great circles; having regard to this uncertainty, the agreement may be regarded as good, region VII being the only one showing important discordance. The surprisingly close agreement of the drift I directions may make us fail to appreciate the definite agreement shown by the drift II directions. If from seventeen points, distributed uniformly all over the Earth, tracks (great circles) were drawn, every one of which passed across the Sahara, they might fairly be considered to show strong evidence of convergence; the distribution of the drift II directions is quite analogous.

The actual determination of the apices and velocities of the drifts was made as follows:—

Each determination of $k\mathrm{V}$ and θ gives two equations of condition to determine the rectangular components X, Y, Z of the drift motion, viz.—

$$k\mathrm{V}\cos\theta = -\mathrm{X}\sin\alpha + \mathrm{Y}\cos\alpha$$
$$k\mathrm{V}\sin\theta = +\mathrm{X}\cos\alpha\sin\delta + \mathrm{Y}\sin\alpha\sin\delta - \mathrm{Z}\cos\delta$$

where α and δ are the R.A. and Dec. of the centre of the region, I found it of interest to solve separately for (1) the equatorial belt Dec. $+36°$ to $-36°$, and (2) the two polar caps above Dec. $+36°$ and below Dec. $-36°$. The areas embraced are approximately as 10 : 7.

If the suffixes 1 and 2 refer to drift I and drift II the results are—

(1) *From the* 10 *Equatorial Regions.*

$X_1 = -\ ·061$	$X_2 = +\ ·109$	Weight 5·4
$Y_1 = +1·462$	$Y_2 = -\ ·370$	,, 5·4
$Z_1 = -\ ·367$	$Z_2 = -\ ·779$	,, 9·1

(2) *From the* 7 *Polar Regions.*

$X_1 = +\ ·018$	$X_2 = +\ ·119$	Weight 5·7
$Y_1 = +1·471$	$Y_2 = -\ ·343$	,, 5·7
$Z_1 = -\ ·441$	$Z_2 = -\ ·731$	,, 2·5

From the whole Sphere.

$X_1 = -\ \cdot020 \pm \cdot026$ $\qquad$ $X_2 = +\ \cdot114 \pm \cdot021$

$Y_1 = +1\cdot467$,, $\qquad$ $Y_2 = -\ \cdot356$,,

$Z_1 = -\ \cdot383$,, $\qquad$ $Z_2 = -\ \cdot769$,,

The comparatively small weight of the determination of Z from the polar area is a well-known fact.

These lead to the following values of the speeds and directions of the drifts, the speeds being measured in terms of the usual theoretical unit $1/h$.

	Drift I.			Drift II.		
	Apex.		Velocity.	Apex.		Velocity.
	R.A.	Dec.		R.A.	Dec.	
10 Equatorial Regions	92°·4	−14°·1	1·507	286°·5	−63°·6	·869
7 Polar Regions .	89°·3	−16°·7	1·536	289°·1	−63°·6	·816
Whole Sphere	90°·8	−14°·6	1·516	287°·8	−64°·1*	·855

The quantities $X_1 - X_2$, $Y_1 - Y_2$, $Z_1 - Z_2$ give the motion of one drift relative to the other. From them we derive—

True vertex, R.A. 94°·2, Dec. +11°·9.

Relative velocity of the drifts, 1·868.

The true vertex falls exactly in the galactic plane.

To find the solar motion it is necessary to know the proportion in which the stars are divided between the two drifts. The mean result from the 17 regions is that 59·6 per cent. belong to drift I and 40·4 per cent. to drift II. From this is determined—

Solar apex, R.A. 267°·3, Dec. +36°·4.

Professor Boss found from the same catalogue,

Solar apex, R.A. 270°·5, Dec. +34°·3.

The agreement is interesting, seeing that the two determinations are made by completely different methods; moreover, Boss's result depends on the magnitudes as well as the directions of the proper motions, whereas in the present investigation only the directions of proper motion are employed.

The speed of the solar motion is 0·908 in terms of the unit $1/h$ employed throughout this paper for measuring the drift

* The paradoxical result that the declination determined from the whole sphere does not lie between the declinations determined from the two parts is due to the unequal weights of the determinations of Z from the two parts.

motions. As the solar motion has been measured spectroscopically in ordinary units, this result would enable us to transform the drift speeds into kilometres per second; but at present this seems inadvisable, in view of the marked discordance between the positions of the solar apex arrived at from the radial motions and the proper motions respectively.

From the definitive results given above, I have calculated the values of the velocity hV and direction θ for each region, and have compared them with the observed values in the following tables. If Q_1 Q_2 be the apices of the two drifts, and P the centre of the region, the angle PQ_1 or PQ_2, is given in the second column of these tables: owing to foreshortening, the drift-velocity in any region should vary as the sine of this angle.

Comparison of the Calculated and Observed Constants of the Drifts.

DRIFT I.

Region.	$\angle PQ_1$.	hV_1.			θ_1.		
		Obs.	Calc.	O − C.	Obs.	Calc.	O − C.
I.	105°	1·65	1·47	+ ·18	0°	359°	+ 1°
II.	102	1·45	1·49	− ·04	10	9	+ 1
III.	70	1·55	1·42	+ ·13	60	58	+ 2
IV.	70	1·30	1·42	− ·12	120	120	0
V.	101	1·50	1·49	+ ·01	170	170	0
VI.	137	0·85	1·04	− ·19	225	224	+ 1
VII.	137	1·05	1·03	+ ·02	320	315	+ 5
VIII.	78	1·45	1·48	− ·03	20	19	+ 1
IX.	48	1·55	1·13	+ ·42	40	39	+ 1
X.	32	1·00	0·80	+ ·20	75	89	− 14
XI.	47	1·05	1·11	− ·06	140	140	0
XII.	77	1·20	1·48	− ·28	165	160	+ 5
XIII.	110	1·25	1·42	− ·17	170	170	0
XIV.	145	0·75	0·88	− ·13	185	179	+ 6
XV.	177	0·15	0·06	+ ·09	255	252	*
XVI.	146	1·00	0·85	+ ·15	355	1	− 6
XVII.	112	1·45	1·41	+ ·04	5	2	+ 3

* Owing to the smallness of hV in this region, θ is hardly determinate, and the comparison has very little significance.

DRIFT II.

Region.	$\angle PQ_2$.	hV_2.			θ_2.		
		Obs.	Calc.	O – C.	Obs.	Calc	O – C.
I.	154°	·45	·37	+ ·08	160°	162°	– 2
II.	127	·80	·68	+ ·12	120	122	– 2
III.	151	·20	·40	– ·20	130	132	– 2
IV.	165	·20	·23	– ·03	45	70	*
V.	141	·40	·55	– ·15	55	49	+ 6
VI.	120	·80	·74	+ ·06	80	68	+12
VII.	114	·75	·77	– ·02	75	96	–21
VIII.	105	·70	·82	– ·12	120	117	+ 3
IX.	121	·80	·73	+ ·07	110	114	– 4
X.	131	·75	·64	+ ·11	115	100	+15
XI.	131	·60	·64	– ·04	80	79	+ 1
XII.	121	·75	·73	+ ·02	55	66	–11
XIII.	105	·90	·83	+ ·07	60	63	– 3
XIV.	91	·85	·85	·00	75	69	+ 6
XV.	82	·95	·85	+ ·10	90	83	+ 7
XVI.	82	·90	·85	+ ·05	100	98	+ 2
XVII.	91	·80	·85	– ·05	105	111	– 6

* Owing to the smallness of hV in this region θ is hardly determinate, and the comparison has very little significance.

The principal discordances are those of hV_1 in region IX and of θ_2 in region VII; region X is altogether rather discordant, all four quantities being somewhat in error. With these exceptions the residuals seem to be of the order of magnitude of the uncertainties of the determination.

We infer that in the main the distribution of the stellar motions follows the same law, quantitatively as well as qualitatively, in all parts of the heavens; and that local deviations and irregularities are comparatively small, being quite insufficient to mask the general law of distribution.

There is a curious tendency shown by the positive residuals of hV_1 (the speed of drift I) to congregate towards a certain belt, the negative ones accumulating at the poles of this belt. The belt coincides roughly with the galactic plane, and may perhaps be identified with it. A similar slight progression in the values of hV_1 was found by Hough and Halm, who, however, showed that it could be attributed to the large dimensions of their regions. Possibly the same explanation holds for my result, although my regions were smaller; but I am more inclined to connect it with the prevalence of type I stars near the galactic plane. The exact connection between spectral type and stellar motion has not yet been made out, but there is considerable evidence that the peculiar

motions of type I stars are less than those of type II; this would increase the value of h, and therefore of hV, for them.

I find no systematic feature in the residuals of $h\mathrm{V}_2$; and no indication of the increase of $h\mathrm{V}_2$ with galactic latitude, which Hough and Halm found.

The Numbers of Stars in the Two Drifts.

The analysis (at the foot of the tables, pp. 10–26) gave the number of stars of each drift for each region. No importance can be attached to these *absolute* numbers, for the "plenitude" of the catalogue is not constant. The *ratio* of the number of stars of drift II to the number of stars of drift I is given in the following table. The regions are arranged in order of galactic latitude. (No sign can be attributed to the galactic latitude, since each region consists of two antipodal parts of the sphere.)

Division of the Stars between the Drifts.

Region.	Number of Stars.	Proportion Drift II : Drift I.	Galactic Latitude.
III.	304	60	1°
X.	356	·66	1
VII.	448	·87	9
II.	354	·48	12
XVI.	311	·69	14
XV.	285	·71	17
I.	371	·60	27
IX.	275	·75	29
XI.	365	·42	31
IV.	294	·87	33
XVII.	245	·77	37
VIII.	308	·67	44
VI.	294	·52	46
XIV.	259	·68	48
XII.	342	·64	61
V.	274	1·03	66
XIII.	237	·59	78

The mean ratio of the number of stars in drift II to drift I is ·68 : 1; or 59·6 per cent. belong to drift I and 40·4 per cent. to drift II. The ratio varies irregularly in the different regions.

It will be noticed that there is a disagreement between these results and those obtained by Hough and Halm. They found an intimate relation between drift II and the galaxy, the proportion of drift II stars being considerably greater in the galactic plane and the speed of drift II being smaller. The results of this investigation show no indication of such a tendency.

The present paper is not concerned with the other very important irregularity of distribution of the drift II stars found by Hough and Halm, viz. a tendency to crowd towards one part of the sphere and away from the opposite. This irregularity was accepted on much stronger evidence than the other; it was supported by the radial motions; and, by the recognition of it, important discordances in determinations of the constant of precession were reconciled. The evidence of Boss's catalogue on this matter can only be obtained by comparing opposite regions of the sky, which has not been attempted in the present paper; meanwhile there is no reason to doubt the existence of this phenomenon.

5. *Stars of the Orion Type.*

At an early stage in this investigation Professor Dyson kindly sent me his copy of Boss's catalogue in which the types of spectrum of the stars had been entered, so far as they are given in *Harvard Annals*, vol. 50. This comprises the great majority of the stars, the number of unknown spectrum being, I think, less than 10 per cent. Using this, I subdivided the stars in each region according to their spectral types. This revealed some remarkable aggregations of the *Orion* (type B) stars moving in nearly the same direction, which clearly called for examination. As regards the other types, although there are doubtless some differences in the distribution of the motions, nothing very striking was shown, and a more minute examination has been deferred.

As is well known, the *Orion* stars are found principally near the galactic plane; in regions near the galactic poles they are almost entirely wanting. The regions which showed the most extensive associations of *Orion* stars were the southern parts of regions II and III, that is, the area R.A. 10^h to 18^h, Dec. $-36°$ to $-70°$.

In region II (south part), out of a total of 62 *Orion* stars, 43 were moving between $\theta = 10°$ and $40°$.

Of the remaining types, only 37 out of 162 were moving between the same limits of θ.

In region III, out of 65 *Orion* stars, 42 were moving between $\theta = 40°$ and $70°$.

Of the remaining stars, 45 out of 135 were moving between the same limits.

When the magnitudes of the proper motions were compared, it became clear that this community of motion was not analogous to the drift motions, but indicated a "moving cluster" similar to the Taurus stream, although more vaguely defined. There was an evident community in the size as well as in the directions of motion, a fact which indicates that the stars form a compact aggregation, and are not distributed at all distances along the line of sight. The proper motion of the cluster is about $4''{\cdot}8$ per century; about 90 stars seem to belong to it—a considerable fraction of the whole number of *Orion* stars (about 700) in the

catalogue. Its motion is directed towards the solar antapex, and its *convergent point* (which cannot be calculated with any great accuracy) seems to be the antapex. This is in accordance with Kapteyn's result that, after abstracting the solar motion, the residual velocities of the *Orion* stars are very small: at the time, however, when the existence of this cluster attracted my attention, the very remarkable evidence on this point given by Kapteyn and Frost (*Astrophysical Journal*, 1910 July) had not been published.

Another smaller but more definite moving cluster of *Orion* stars is described in a separate paper.

The smallness of the proper motions of the stars of this type is well known. It cannot be attributed to the smallness of their peculiar motions, because parallactic as well as peculiar motion goes to make up the resultant proper motion. It must indicate that the majority of these stars are very remote from the Sun compared with the other types. When the stars of the moving clusters are removed, this smallness becomes particularly noticeable. It occurred to me that by removing the *Orion* stars altogether from the investigation of the two star-drifts I should not only get rid of the moving clusters—a very necessary step—but I should also eliminate the majority of the very small proper motions of the catalogue, a step which should considerably increase the accuracy of the results. The directions of the smallest proper motions cannot be determined with much accuracy. They are retained because the mathematical analysis requires that the proper motions should not be selected with reference to size; but there is no objection to our removing from the analysis the whole class of stars which furnishes the great majority of these small proper motions.

6. *Deviations from the Two-drift Distribution.*

The comparatively small number of stars in each region precludes our attaching importance to the slight differences between the calculated and observed distributions shown in the tables. But it is of interest to examine whether there is anything systematic in the occurrence of these residuals. The only tendency of this sort that I can trace is an excess of positive residuals (observed – calculated) towards the solar apex. In figs. 2 to 9 there seems to be a bulge in the curves in directions roughly towards the solar antapex (indicated by the broken arrows); the bulge is generally rather displaced towards the direction of drift II. The tendency is much more strongly shown in regions XIV and XVI, for which the diagrams (not reproduced) almost present the appearance of a three-drift distribution.

There seems to be a small extra stream in this direction, which, however, is rather hard to define. It is natural to associate it with the peculiarity of the *Orion* type stars, which, according to Kapteyn and Frost, have little motion except the solar motion. The *Orion* stars were excluded from this investi-

gation, but it may well be that the few stars forming this subsidiary stream are associated with them in this particular: the correlation between spectral type and any peculiarity of motion is not likely to be absolute; and in removing the *Orion* stars we should not expect to eliminate entirely a phenomenon which they show strongly.

The third stream is an insignificant phenomenon compared with the two great star-drifts, but it seems to prevail over a large part of the sky, and is worth some attention.

7. *Comparison with other investigations.*

The principal determinations of the directions of the two streams are as follows:—

Apices of the two Drifts.

Catalogue.	Investigator.	Drift I.		Drift II.	
		R.A.	Dec.	R.A.	Dec.
Bradley	Kapteyn	85°	−11°	260°	−48°
,,	Hough and Halm	87	−13	276	−41
Large proper motions	Dyson	93	− 7	246	−64
Groombridge	Eddington	90	−19	292	−58
Boss	,,	91	−15	288	−64

True Vertex.

Catalogue.	Investigator.	R.A.	Dec.	Theory.
Bradley	Kapteyn	91°	+13°	Two-stream.
,,	Rudolf	96	+ 6	Ellipsoidal.
Groombridge	Eddington	95	+ 3	Two-drift.
,,	Schwarzschild	93	+ 6	Ellipsoidal.
Boss	Eddington	94	+12	Two-drift.
Large proper motions	Dyson	88	+21	,,
,,	Beljawski	86	+24	Ellipsoidal.
Radial motions	Hough and Halm	88	+27	Two-drift.

The results from my investigation of Zodiacal Stars are discordant for an unexplained reason, and have not been included in the above table.

The results from Boss's catalogue are fairly central between the other determinations. The determinations of the true vertex seem to divide themselves into two groups rather distinctly, but the division may be partly accidental.

The work of Dyson and Hough and Halm has indicated that the speed of drift II, which I found from the Groombridge proper motions, was too low; the present research leads to a higher value. The ratio of the speeds of the two drifts 1·52 : ·86 is in fair accordance with the result 3 : 2 found by Dyson from the very large proper motions.

The disparity between the numbers of stars of the two drifts is not quite so great as that given by the results of Hough and Halm. It is possible that the proportion of drift II stars may be increased by including fainter stars; if this is so, it would also account for the greater proportion of drift II stars found in the Groombridge catalogue.

8. *Summary of Results.*

1. The phenomena of two star-streams are well shown in Professor Boss's "Preliminary General Catalogue." This is illustrated in figs. 1–9.

2. The analytical investigation based on the two-drift theory leads to the following results:—

	R.A.	Dec.	Velocity.
Motion of Drift I	90°·8	−14°·6	1·52
,, ,, ,, II	287°·8	−64°·1	·86
True vertex	94°·2	+11°·9	1·87
Solar motion	267°·3	+36°·4	·91

3. The stars are distributed between the two drifts in the proportion 3 of drift I to 2 of drift II. This ratio varies somewhat irregularly in different parts of the sky.

4. The results of the present investigation are antagonistic to the view that there is an intimate connection between drift II and the galaxy.

5. The stars of the *Orion* type show marked peculiarities. Those which are nearest to our system seem to aggregate into clusters for the most part. Most of the remainder are extremely distant.

Note on a Moving Cluster of Stars of the Orion type in Perseus. By A. S. Eddington, M.A., M.Sc.

(*Communicated by the Astronomer Royal.*)

The diagram represents the proper motions of all the stars of type B (*Harvard Annals*, vol. 50) given in Professor Boss's "Preliminary General Catalogue" between R.A. 2^h and 6^h and Dec. $+36°$ and $+70°$. Each cross indicates the proper motion of a

Moving Cluster of "Orion" Stars in Perseus.

star, the star moving from the origin O to the cross in a century. If all the stars were to start from the origin at the same instant, with their actual proper motions, then after the lapse of a century they would be distributed as in the diagram. One star only has travelled beyond the limits of the diagram and is not shown.

The crosses form two widely separated clusters. The upper group, which is close to the origin, consists of stars with very minute proper motions, all less than 1″·5 per century. These are presumably very remote stars, not necessarily associated with one another; they only appear to cling together because the great distance renders their diverse motions inappreciable.

The lower group consists of 16 stars sharing very nearly the

same motion both as regards magnitude and direction. They evidently form a moving cluster similar in character to the Taurus stream or Ursa Major stream. Their association is further confirmed by the fact that they are not scattered over the whole area of the sky investigated, but occupy quite a limited region of it.

Particulars of the 16 members are given in the following table:—

Moving Cluster in Perseus.

Boss's No.	Name of Star.	Mag.	R.A. h m	Dec. °	Centennial Motion. ″	Direction. °
740	30 Persei	5·5	3 11	+ 44	3·8	55
742	29 Persei	5·3	3 12	+ 50	4·5	52
744	31 Persei	5·2	3 12	+ 50	4·2	51
767	Pi. 37	5·4	3 16	+ 49	3·6	45
780	Brad. 476	5·1	3 21	+ 49	3·2	47
783	Pi. 56	5·8	3 22	+ 50	4·9	37
790	34 Persei	4·8	3 22	+ 49	4·4	57
796	Brad. 480	6·1	3 24	+ 48	4·7	54
817	ψ Persei	4·4	3 29	+ 48	4·3	42
838	δ Persei	3·0	3 36	+ 47	4·6	51
898	Pi. 186	5·5	3 49	+ 48	3·9	40
910	ϵ Persei	2·9	3 51	+ 40	3·9	49
947	*c* Persei	4·2	4 1	+ 47	4·4	43
1003	*d* Persei	4·9	4 14	+ 46	4·5	55
1253	15 Camelopardi	6·4	5 11	+ 58	3·5	37
1274	ρ Aurigæ	5·3	5 15	+ 42	4·5	40

In the last column the "direction" is the angle between the direction of motion and the meridian of 4 hours R.A.

The stars are arranged in a kind of chain, and not in the form of a globular cluster.

For one star, *d* Persei, the radial velocity has been published by Kapteyn and Frost, viz. + 4 km. per sec.

The cluster is not moving towards the apex of either drift or towards the solar antapex. It has an individual motion of its own, just as the other moving clusters have. The convergent point might perhaps be determined, but the result would necessarily be very uncertain.

Third Note on the number of Faint Stars with large Proper Motions. Zone +27°. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In November 1908 the results of a comparison of ten pairs of plates in zone +28°, taken at intervals of 10 to 15 years, and in April 1909, the comparison of 30 plates in zone +26°, were communicated to the Society. The present note gives a similar comparison for 29 plates in zone +27°. A comparison for zone +25° is about half done, and will complete the examination of material available at present. A sum of £100 has been kindly placed at the disposal of the University Observatory for this special investigation by the Government Grant Fund.

But when the straightforward computations for comparison of measures have been completed, there remains the careful examination and remeasurement of all exceptional cases which may be large proper motions. For this work I am in the present instance indebted to the volunteer help of Mr. W. H. Waterfield, B.A., of Christ Church.

Table I. which follows corresponds to Table I. in the previous papers. As a rule, cases have been specially examined where the differences exceeded ±1″·2 in either coordinate, and the figures in column 4 give the number of cases where these differences were confirmed on remeasurement, and thus probably indicate a P.M., although no stress is laid on these records unless the P.M. exceeds 15″ a century.

TABLE I.

Zone +27°. Stars having probable large P.Ms.

R.A. of Plate Centre. h m	Gal. Lat. °	Total Stars compared.	Interval. y	Total P.Ms.	Centennial P.Ms. 15″–20″.	Centennial P.Ms. >20″.
3 16	22	124	12·2	3	0	2
3 32	20	150	12·1	2	0	1
3 56	16	184	12·1	2	1	1
4 4	15	125	12·2	4	1	2
4 12	14	33	12·2	0	0	0
4 28	10	48	10·0	3	2	1
4 44	7	54	12·2	2	2	0
4 52	3	53	10·0	0	0	0
5 0	0	75	14·1	4	2	1
5 8	3	87	12·1	5	1	2
5 16	2	75	14·2	1	1	0
5 24	0	94	10·0	0	0	0
7 8	17	172	14·1	5	2	0
7 16	20	134	14·0	3	1	1
7 56	28	115	14·0	1	0	0

TABLE I.—*continued.*

R.A. of Plate Centre. h m	Gal. Lat. °	Total Stars compared.	Interval. y	Total P.Ms.	Centennial P.Ms. 15″–20″.	P.Ms. > 20″.
8 4	31	130	14·0	0	0	0
8 12	33	137	14·1	3	0	1
8 52	42	135	14·2	6	2	2
9 8	44	118	14·2	5	3	2
10 36	63	108	10·0	3	0	2
12 20	84	90	14·0	3	1	1
13 40	76	79	9·9	3	1	2
14 12	71	95	14·0	2	0	1
14 20	68	93	14·0	3	1	0
14 52	63	104	14·0	5	1	2
16 20	42	110	9·9	1	1	0
19 40	2	516	15·0	4	1	1
21 48	19	246	15·0	5	2	2
22 4	24	267	15·0	6	2	0
29 plates	...	3751	...	84	28	27
40 plates in +28° & +26°		9332	...	151	42	26
69 plates in all		13083	...	235	70	53

3. The fact brought out by former results (see vol. lxix. p. 492), that the crop of large P.Ms. is not much greater for a plate in the galaxy than for plates away from it, is confirmed by these independent results. Arranging the plates simply according to the number of stars shown we get—

TABLE II.

Plates in Order of Number of Stars.

No. Stars.	P.Ms. 15″–20″.	P.Ms. > 20.″	No. Stars.	P.Ms. 15″–20″.	P.Ms. > 20″.	No. Stars.	P.Ms. 15″–20″.	P.Ms. > 20″.	No. Stars.	P.Ms. 15″–20″.	P.Ms. > 20.″
516	1	1	150	0	1	108	0	2	54	2	0
267	2	0	137	0	1	104	1	2	53	0	0
246	2	2	135	2	2	95	0	1	48	2	1
184	1	1	134	1	1	94	0	0	33	0	0
172	2	0	130	0	0	93	1	0			
			125	1	2	90	1	1			
Sums below multiplied by 2·0 for comparison.			124	0	2	87	1	2	Sums below multiplied by 2·5 for comparison.		
			118	3	2	79	1	2			
			115	0	0	75	2	1			
			110	1	0	75	1	0			
[2770]	[16]	[8]	1278	8	11	900	8	11	[470]	[10]	[3]

4. But it is perhaps more appropriate to arrange the plates in order of galactic latitude, and we may do this for all three zones together. The results are as follows:—

Table III.

No. of Plates.	Mean Gal. Lat.	Mean Number of P.Ms. per Plate.			Mean No. of Stars on Plate.
		In all.	15″–20″.	> 20″.	
13	4°	2·9	1·1	0·7	293
11	15	4·4	1·5	0·8	268
13	23	2·9	0·9	0·8	209
11	32	2·0	0·6	0·6	170
11	53	2·8	1·0	1·1	106
10	78	3·7	0·9	0·6	68

5. It seems clear that stars with P.Ms. greater than 15″ per century (and probably the limit can be reduced) are scattered with no sensible reference to the galaxy, and the inference is that whatever causes the galaxy lies beyond them.

6. There are on the average 1·0 stars per plate (about 2° × 2°) with P.M. between 15″ and 20″ per century, and 0·8 stars per plate with P.M. > 20″. So that on the whole sphere we may expect about 10,000 stars with P.M. between 15″ and 20″, and 8000 > 20″, among the stars of magnitude (say) 10·5 and brighter. The limiting magnitude is a little indefinite, since the comparison deals with plates which were admittedly less rich in stars than they should have been; but we may put the whole number at about 2 or 3 million.

7. The following is a list of stars having P.M. greater than 15″ per century on these 29 plates in zone +27°. It may not have been noticed that the Oxford numbers in the second column give the means of obtaining the equatorial coordinates of the star in the sky when the reductions indicated in the volumes of the *Oxford Astrographic Catalogue* are carried out. The measured coordinates are first to be corrected by the constants given at the head of each plate, and then reduced to equatorial coordinates by the tables given in the introduction to each volume.

[Table.

TABLE IV.

List of Large P.Ms. in Zone +27°.

R.A. of Plate Centre. h m	Oxford Number.	Cent P.M. in Arc.	Δx.	Δy.	Interval.	Mag.
		″			yr	
8 52	23081*	39·2	+ 4·5	−18	14·2	7·0
8 12	31703*	35·1	+ 0·5	−16·5	14·1	5·7
14 52	32902*	32·3	− 8·5	−12·5	14·0	9·5
5 0	10504	30·6	+ 6	−13	14·1	11·3
9 8	24000	29·9	−12·5	− 7	14·2	10·2
14 52	33053	29·1	−13·5	− 2	14·0	10·1
3 56	8844	27·8	+11	+ 2	12·1	11·3
13 40	31117	27·3	− 9	0	9·9	[12·0]
5 8	10665*	26·8	+10	− 4	12·1	7·3
19 40	52491*	25·2	− 1·5	+12·5	15·0	7·1
21 48	64737*	25·2	−10·5	− 7·5	15·0	6·8
9 8	24044	24·8	+ 9·5	− 7	14·2	9·6
5 8	10670*	24·8	+10	− 1	12·1	8·9
10 36	26534*	24·3	+ 8	− 1·5	10·0	8·4
3 32	7890	24·0	+ 5·5	− 8	12·1	11·0
7 16	17930	23·9	+ 2·5	+11	14·0	10·0
12 20	29343*	23·9	+ 4	−10·5	14·1	8·0†
8 52	23050*	23·5	− 5	−10	14·2	9·1
3 16	7175	23·0	+ 8	− 5	12·2	11·0
3 16	7228	23·0	+ 8	− 5	12·2	10·0
4 28	9707*	21·9	+ 7	− 2	10·0	8·6
21 48	64113*	21·4	+10·5	+ 2	15·0	8·8
14 12	32030	20·5	− 1·5	− 9·5	14·0	9·7
4 4	9148	20·4	+ 2	− 8	12·2	10·9
4 4	9149	20·4	+ 2	− 8	12·2	10·9
10 36	26431*	20·1	− 4·5	+ 5	10·0	8·9
13 40	31102	20·0	− 6·5	− 1	9·9	10·4
16 20	35320	20·0	− 1	+ 6·5	9·9	11·2
21 48	64188	19·4	− 9·5	− 2	15·0	10·8
5 8	10614*	19·4	− 5	− 6	12·1	7·0
3 56	8968	19·3	+ 7·5	− 2	12·1	10·6
7 16	18449*	19·2	+ 7·5	− 5	14·0	8·1
13 40	31145	19·1	− 2	− 6	9·9	11·7
14 20	32211	18·2	− 8	+ 3	14·0	[12·0]
22 4	65954	18·0	+ 9	0	15·0	10·3

† A double star ; mags. 8·0 and 8·2, distance 4″, measured as one mass on the Oxford plates.

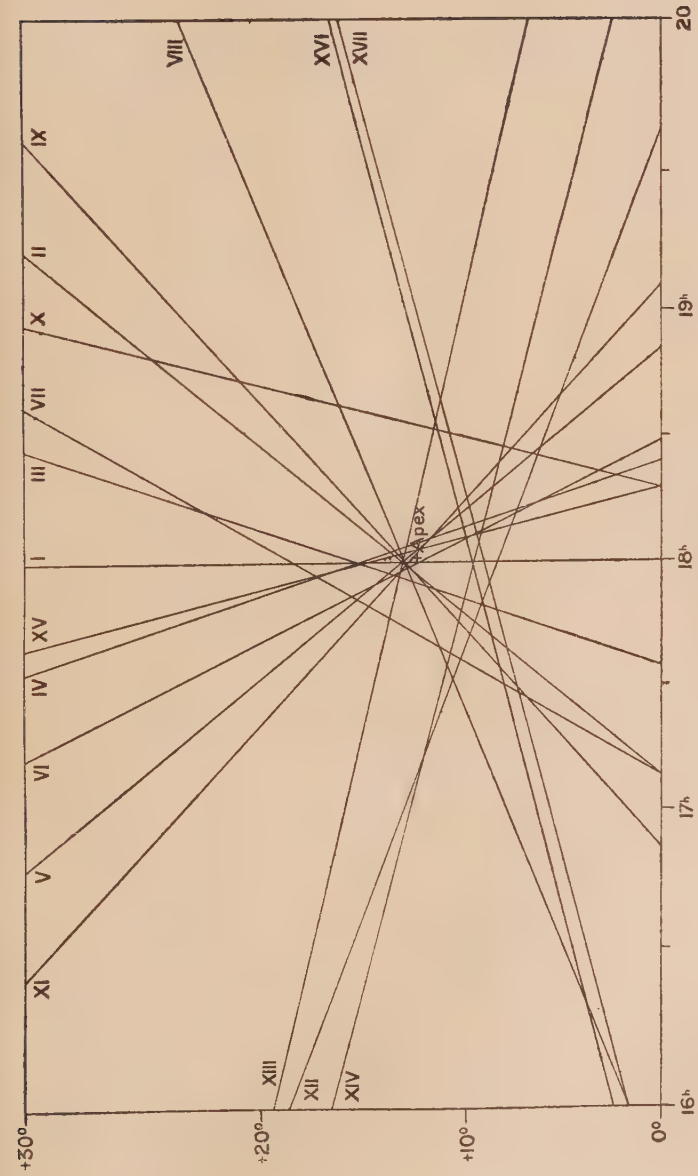


FIG. 12.—CONVERGENCE OF THE DIRECTIONS OF DRIFT I FROM THE 17 REGIONS.

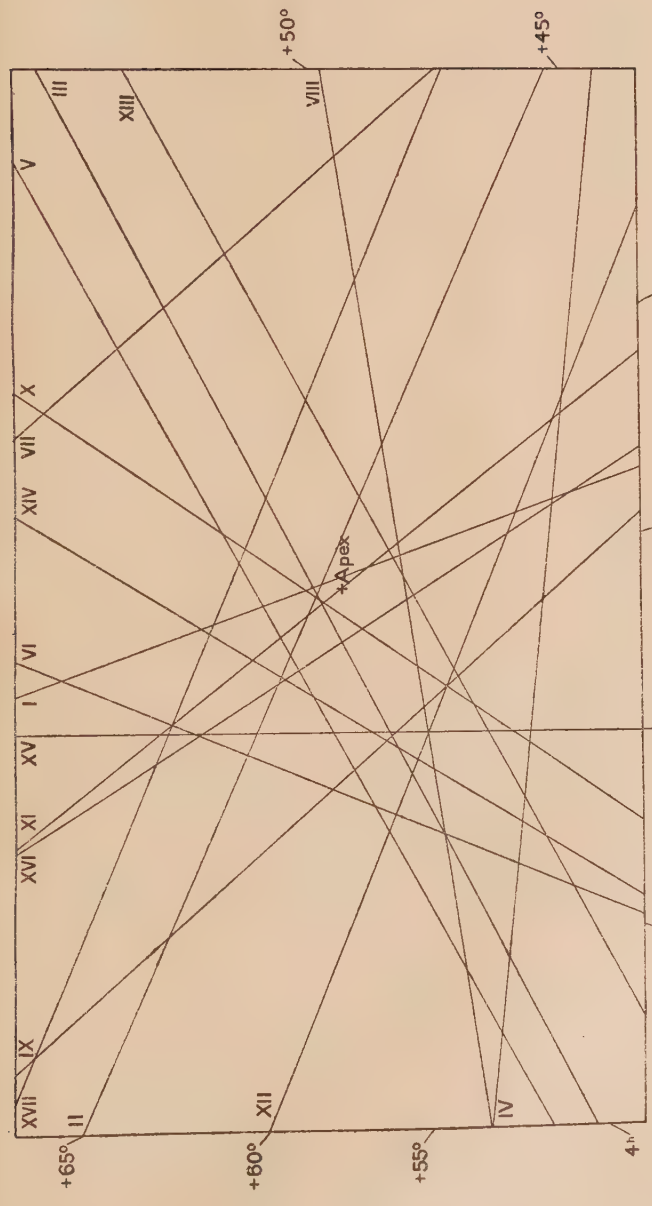


FIG. 13.—CONVERGENCE OF THE DIRECTIONS OF DRIFT II FROM THE 17 REGIONS.

TABLE IV.—*continued.*

R.A. of Plate Centre. h m	Oxford Number.	Cent. P.M. in Arc.	Δx.	Δy.	Interval. yr	Mag.
		″				
4 44	10214	17·8	+ 1·5	− 7	12·2	10·3
4 4	9109*	17·7	− 4	− 6	12·2	7·6
8 52	23037	17·2	− 8	− 1	14·2	11·8
4 28	9751	17·1	+ 5·5	− 1·5	10·0	10·4
5 0	10517*	17·0	+ 8	0	14·1	7·3
7 8	17408	17·0	+ 8	0	14·1	11·2
12 20	29299	16·6	− 6	+ 5	14·1	11·3
4 44	10103*	16·5	+ 5	− 4·5	12·2	9·2
4 28	9758	16·5	0	− 5·5	10·0	11·2
21 48	64622*	16·2	+ 8	− 1	15·0	9·0
5 16	10848	16·0	+ 7	− 3	14·2	10·0
22 4	65836*	16·0	0	− 8	15·0	8·0
5 0	10523*	15·5	+ 7	− 2	14·1	8·9
19 40	51969*	15·4	− 1·5	− 7·5	15·0	9·2
14 52	33072	15·4	− 7	+ 1·5	14·0	11·2
7 8	17662*	15·3	+ 6	− 4	14·1	8·1
8 52	23380	15·1	+ 1	− 7	14·2	11·8
9 8	23785	15·0	− 7	+ 2	14·2	11·6
9 8	23786*	15·0	− 7	+ 2	14·2	9·1
9 8	23811	15·0	− 7	+ 2	14·2	9·9

The stars marked with an asterisk are in the Cambridge Catalogue, and are therefore comparatively bright stars; it will be seen that there are 24 out of 55. [In the last note the numbers were 24 out of 50.] The magnitudes for these stars are simply those of the Cambridge Catalogue. For the other stars the magnitudes are inferred from the formulæ in the printed volume for + 27°, but are to be taken as provisional only.

8. If, now, we group these stars according to magnitude, we get results as follows, N denoting the *total* number as bright as, or brighter than, the reading.

TABLE V.

Showing P.Ms. for successive Magnitudes.

Mag.	To 7·9.	8·9.	9·9.	10·9.	12·0.
N over 20″	5	11	15	22	28
N 15″–20″	3	7	12	18	27
N over 15″	8	18	27	40	55
log N over 15″	0·90	1·26	1·43	1·60	1·74
Diff.		0·36	0·17	0·17	0·14
From last paper		0·36	0·22	0·16	0·19

Summary.

On 69 plates in zones $+26°$, $+27°$, and $+28°$, containing altogether 13,000 stars, there are 123 stars with P.M. exceeding 15″ per century, of which 53 have P.M. exceeding 20″ per century. There are also 112 other stars with P.Ms. which are smaller, but fairly discernible with the rather rough methods here adopted. The large P.Ms. are scattered uniformly in R.A., with no condensation towards the Milky Way, and thus probably indicate stars belonging to our Solar Cluster.

The motion of Jupiter's Eighth Satellite from 1910 *to* 1916.

By A. C. D. Crommelin, D.Sc. (Plate 3.)

(*Communicated by the Astronomer Royal.*)

This paper is in continuation of that by Mr. Cowell, Mr. Davidson, and the writer in *Monthly Notices*, vol. lxix., No. 5. The motion has been carried on by the same method and in the same orbit as that used previously. It is evident that the orbit needs some small adjustment; but as it is still too soon to reach an absolutely final orbit, this adjustment has been postponed till the satellite has been observed over several revolutions. The various tables given are similar to those in the former paper. The equator and equinox of 1910·0 are used throughout. The coordinates of Jupiter for 1913–1914 are taken from the *Annuario Astronomico pel* 1910, *pubblicato dal R. Osservatorio di Torino.* Those for 1915 have been carried on by an approximate method, great accuracy being unnecessary for this purpose, since the solar perturbing action is small compared to Jupiter's action.

The geocentric places are true places for Paris mean noon, the equation of light not having been applied.

The axis of x turns to the First Point of Aries, that of y to R.A. 6^h, Dec. $0°$, that of z to the North Pole of the Equator.

An examination has been made of these coordinates to ascertain the values of sidereal period, eccentricity, inclination, node, etc. The results below are computed in most cases with four-figure logarithms; they are therefore much less accurate than the coordinates themselves, and these latter should be used in all cases where accuracy is required.

The Sidereal Period.

This has been determined by the interval between the passages through the planes $x=0$, $y=0$, $x=\pm y$.

Passage through $x=0$ (y dimin.)	Interval. d	Passage through $x=0$ (y increasing)	Interval. d	Passage through $y=0$ (x increasing)	Interval. d	Passage through $y=0$ (x dimin.)	Interval. d
1908 May 21		1908 Dec. 23		1908 Oct. 5		1909 Aug. 10	
	761		742		752		723
1910 June 21		1911 Jan. 4		1910 Oct. 27		1911 Aug. 3	
	738		720		713		768
1912 June 28		1912 Dec. 24		1912 Oct. 9		1913 Sept. 9	
	717		781		754		735
1914 June 15		1915 Feb. 13		1914 Nov. 2		1915 Sept. 14	
Mean	738·7		747·7		739·7		742·0

Passage through $x=-y$ ($x+$, $y-$)	Interval. d	Passage through $x=y$ ($x-$, $y-$)	Interval. d	Passage through $x=-y$ ($x-$, $y+$)	Interval. d	Passage through $x=y$ ($x+$, $y+$)	Interval. d
1908 Jan. 20		1908 Aug. 18		1908 Nov. 8		1909 Mar. 27	
	720		762		747		726
1910 Jan. 9		1910 Sept. 19		1910 Nov. 25		1911 Mar. 23	
	760		715		716		737
1912 Feb. 8		1912 Sept. 3		1912 Nov. 10		1913 Mar. 29	
	734		738		765		722
1914 Feb. 11		1914 Sept. 11		1914 Dec. 15		1915 Mar. 21	
	722						
1916 Feb. 3							
Mean	734·0		738·3		742·7		728·3

Mean sidereal period, ... 738·9 days
Mean discordance of above means, . . ±4·1 ,,
Mean synodic period, ... 631·3 ,,
No. of sidereal revolutions in one revolution of Jupiter, 5·864
,, synodic ,, ,, ,, 6·864

Apsidal Passages and Eccentricity.

Perijove.	Interval. d	Distance.	Apojove.	Interval. d	Distance.
1908 Oct. 19		·1002	1909 Dec. 7		·2159
	757			696	
1910 Nov. 15		·0901	1911 Nov. 3		·2277
	702			681	
1912 Oct. 17		·0904	1913 Sept. 14		·2175
	767			865	
1914 Nov. 23		·1111	1916 Jan. 27		·2058
Mean	742	·0980		747	·2167

The mean semi-major axis of the orbit (considered as an ellipse) has been deduced as ·1572; taking this value, the eccentricities at the above dates, reckoned by the formulæ perijove-distance $= a(1-e)$, apojove-distance $= a(1+e)$, are

At perijoves ·3628, ·4269, ·4250, ·2940 Mean ·3772
At apojoves ·3733, ·4480, ·3837, ·3092 Mean ·3785

The distance of satellite IV from Jupiter in astronomical units is ·012583, and its period 16·68902. The distance corresponding to a period 738·9 days in purely elliptical motion about Jupiter is

$$\left(\frac{738\cdot9}{16\cdot689}\right)^{\frac{2}{3}} \times \cdot012583 = \cdot1575.$$

The following are the positions of the ascending node of the orbit of VIII on Jupiter's orbit, and the inclinations of VIII's orbit to Jupiter's orbit. The longitudes are reckoned from the ascending node of Jupiter's orbit on the Earth's equator.

Date.	Longitude of Ascending Node.	Inclination.
1908 July 21	279° 0′	28° 14′
1909 Feb. 9	278 18	29 14
1910 July 27	288 13	34 35
1911 Jan. 21	290 8	34 4
1912 July 17	295 43	33 22
1913 Jan. 7	298 7	32 2
1914 July 8	297 48	31 52
1915 Mar. 1	298 59	32 5

The node advances 20° in $6\frac{2}{3}$ years, or 3° per annum.

The mean of the inclinations is 31° 56′.

The following table gives the dates of opposition and the declinations of Jupiter and VIII.

Date of Opposition in R.A.	Dec. of Jupiter.	Dec. of VIII.
1911 May 1	− 13° 23′	− 14° 52′
1912 June 2	− 21 14	− 20 59
1913 July 6	− 22 56	− 24 7
1914 Aug. 11	− 16 39	− 16 13
1915 Sept. 18	− 4 7	− 5 11

It is doubtful whether VIII will be observable at Greenwich before 1915. Hence it is hoped that the possessors of large photographic telescopes in more southern latitudes will cooperate in following it.

Table I.

Jovicentric Coordinates of Sun and reversed Attractions of Sun on Jupiter in Eighth Decimal Place.

	Jovicentric Coordinates of Sun.			Reversed Attractions of Sun on Jupiter in Eighth Decimal Place.		
Paris Noon. 1910.	*x.*	*y.*	*z.*	X.	Y.	Z.
June 10	+ 5·265 260	+ 1·346 954	+ ·449 157	− 2 459 42	− 629 17	− 209 80
26	5·233 966	1·448 399	·493 479	2 445 20	676 66	230 54
July 12	5·200 226	1·549 178	·537 568	2 430 00	723 92	251 20
28	5·164 050	1·649 228	·581 409	2 413 80	770 88	271 76
Aug. 13	5·125 460	1·748 502	·624 974	2 396 64	817 60	292 24
29	5·084 470	1·846 959	·668 250	2 378 48	864 00	312 60
Sept. 14	+ 5·041 100	+ 1·944 553	+ ·711 211	− 2 359 40	− 910 12	− 332 88

TABLE I.—*continued.*

Paris Noon.	Jovicentric Coordinates of Sun.			Reversed Attractions of Sun on Jupiter in Eighth Decimal Place.		
	x.	*y.*	*z.*	X.	Y.	Z.
1910.						
Sept. 30	+4·995 368	+2·041 233	+·753 841	−2 339 28	−955 88	−353 00
Oct. 16	4·947 297	2·136 961	·796 116	2 318 24	1 001 36	373 04
Nov. 1	4·896 902	2·231 687	·838 017	2 296 24	1 046 48	392 96
17	4·844 211	2·325 360	·879 526	2 273 28	1 091 24	412 72
Dec. 3	4·789 246	2·417 946	·920 619	2 249 32	1 135 60	432 40
19	4·732 028	2·509 391	·961 284	2 224 44	1 179 64	451 88
1911.						
Jan. 4	4·672 584	2·599 657	1·001 492	2 198 60	1 223 24	471 24
20	4·610 936	2·688 704	1·041 232	2 171 84	1 266 44	490 44
Feb. 5	4·547 119	2·776 478	1·080 479	2 144 12	1 309 20	509 48
21	4·481 156	2·862 940	1·119 219	2 115 44	1 351 52	528 36
Mar. 9	4·413 076	2·948 056	1·157 429	2 085 84	1 393 40	547 04
25	4·342 907	3·031 771	1·195 090	2 055 32	1 434 80	565 60
Apr 10	4·270 677	3·114 054	1·232 185	2 023 84	1 475 72	583 92
26	4·196 426	3·194 857	1·268 697	1 991 44	1 516 16	602 08
May 12	4·120 180	3·274 145	1·304 607	1 958 14	1 556 06	620 02
28	4·041 973	3·351 879	1·339 897	1 923 91	1 595 44	637 77
June 13	3·961 844	3·428 016	1·374 543	1 888 77	1 634 28	655 30
29	3·879 823	3·502 512	1·408 541	1 852 73	1 672 55	672 62
July 15	3·795 943	3·575 338	1·441 861	1 815 78	1 710 25	689 71
31	3·710 250	3·646 450	1·474 491	1 777 93	1 747 36	706 57
Aug. 16	3·622 776	3·715 812	1·506 418	1 739 19	1 783 85	723 19
Sept. 1	3·533 566	3·783 388	1·537 617	1 699 56	1 819 72	739 56
17	3·442 649	3·849 143	1·568 075	1 659 05	1 854 95	755 67
Oct. 3	3·350 078	3·913 042	1·597 777	1 617 67	1 889 52	771 53
19	3·255 885	3·975 051	1·626 706	1 575 43	1 923 41	787 11
Nov. 4	3·160 114	4·035 133	1·654 851	1 532 31	1 956 61	802 42
20	3·062 813	4·093 257	1·682 190	1 488 35	1 989 09	817 45
Dec. 6	2·964 019	4·149 390	1·708 712	1 443 54	2 020 84	832 18
22	2·863 783	4·203 501	1·734 398	1 397 89	2 051 85	846 61
1912.						
Jan. 7	2·762 146	4·255 562	1·759 242	1 351 41	2 082 08	860 73
23	2·659 153	4·305 634	1·782 980	1 304 11	2 111 59	874 42
Feb. 8	2·554 861	4·353 394	1·806 324	1 256 00	2 140 19	888 01
24	2·449 308	4·399 216	1·828 293	1 207 09	2 168 06	901 04
Mar. 11	2·342 547	4·442 660	1·849 855	1 157 39	2 194 99	913 96
27	2·234 627	4·484 015	1·870 253	1 106 90	2 221 11	926 41
Apr. 12	+2·125 604	+4·523 141	+1·889 725	−1 055 65	−2 246 35	−938 50

TABLE I.—*continued.*

Paris Noon.	Jovicentric Coordinates of Sun.			Reversed Attractions of Sun on Jupiter in Eighth Decimal Place.		
	x.	*y.*	*z.*	X.	Y.	Z.
1912.						
Apr. 28	+2·015 516	+4·560 022	+1·908 258	−1 003 64	−2 270 69	−950 23
May 14	1·904 432	4·594 629	1·925 839	950 89	2 294 11	961 58
30	1·792 395	4·626 939	1·942 459	897 40	2 316 58	972 54
June 15	1·679 458	4·656 935	1·958 103	843 20	2 338 09	983 10
July 1	1·565 677	4·684 580	1·972 778	788 30	2 358 63	993 27
17	1·451 109	4·709 876	1·986 433	732 71	2 378 16	1 003 01
Aug. 2	1·335 804	4·732 792	1·999 097	676 45	2 396 67	1 012 33
18	1·219 821	4·753 303	2·010 746	619 53	2 414 13	1 021 23
Sept. 3	1·103 222	4·771 401	2·021 375	561 98	2 430 53	1 029 68
19	·986 060	4·787 062	2·030 975	503 81	2 445 85	1 037 68
Oct. 5	·868 397	4·800 280	2·039 531	445 04	2 460 06	1 045 22
21	·750 284	4·811 032	2·047 047	385 69	2 473 15	1 052 30
Nov. 6	·631 785	4·819 315	2·053 502	325 78	2 485 09	1 058 89
22	·512 963	4·825 104	2·058 904	265 34	2 495 86	1 065 00
Dec. 8	·393 878	4·828 399	2·063 238	204 38	2 505 45	1 070 61
24	·274 582	4·829 183	2·066 502	142 93	2 513 84	1 075 72
1913.						
Jan. 9	·155 151	4·827 450	2·068 684	81 02	2 521 00	1 080 31
25 +	·035 543	4·823 199	2·069 780	− 18 63	2 526 92	1 084 37
Feb. 10 −	·083 995	4·816 420	2·069 799	+ 44 15	2 531 56	1 087 91
26	·203 486	4·807 110	2·068 731	107 30	2 534 91	1 090 90
Mar. 14	·322 877	4·795 237	2·066 560	170 82	2 536 99	1 093 35
30	·442 078	4·780 839	2·063 333	234 66	2 537 72	1 095 24
Apr. 15	·561 063	4·763 943	2·058 969	298 80	2 537 09	1 096 53
May 1	·679 720	4·744 464	2·053 520	363 20	2 535 14	1 097 27
17	·798 093	4·722 449	2·046 965	427 88	2 531 82	1 097 43
June 2	·915 914	4·697 909	2·039 325	492 69	2 527 09	1 096 99
18	1·033 379	4·670 824	2·030 572	557 74	2 520 95	1 095 95
July 4	1·150 178	4·641 240	2·020 739	622 86	2 513 40	1 094 30
20	1·266 419	4·609 128	2·009 780	688 12	2 504 41	1 092 03
Aug. 5	1·381 919	4·574 498	1·997 768	753 41	2 493 98	1 089 17
21	1·496 697	4·537 391	1·984 648	818 73	2 482 07	1 085 65
Sept. 6	1·610 661	4·497 783	1·970 440	884 04	2 468 69	1 081 51
22	1·723 701	4·455 711	1·955 155	949 27	2 453 83	1 076 73
Oct. 8	1·835 803	4·411 194	1·938 787	1 014 40	2 437 46	1 071 30
24	1·946 920	4·364 226	1·921 350	1 079 40	2 419 58	1 065 22
Nov. 9	−2·056 957	+4·314 835	+1·902 844	+1 144 21	−2 400 18	−1 058 48

TABLE I.—*continued.*

Paris Noon.	Jovicentric Coordinates of Sun.			Reversed Attractions of Sun on Jupiter in Eighth Decimal Place.		
	x.	*y.*	*z.*	X.	Y.	Z.
1913.						
Nov. 25	−2·165 787	+4·263 054	+1·883 285	+1 208 76	−2 379 28	−1 051 09
Dec. 11	2·273 462	4·208 880	1·862 668	1 273 06	2 356 83	1 043 03
27	2·379 900	4·152 325	1·840 999	1 337 06	2 332 84	1 034 30
1914.						
Jan. 12	2·484 897	4·093 483	1·818 315	1 400 64	2 307 33	1 024 91
28	2·588 551	4·032 311	1·794 594	1 463 82	2 280 26	1 014 84
Feb. 13	2·690 753	3·968 841	1·769 849	1 526 55	2 251 65	1 004 09
Mar. 1	2·791 387	3·903 144	1·744 112	1 588 74	2 221 51	992 68
17	2·890 453	3·835 209	1·717 372	1 650 38	2 189 82	980 58
Apr. 2	2·987 857	3·765 097	1·689 667	1 711 41	2 156 60	967 82
18	3·083 577	3·692 793	1·660 978	1 771 77	2 121 81	954 37
May 4	3·177 485	3·618 411	1·631 339	1 831 39	2 085 52	940 25
20	3·269 578	3·541 908	1·600 760	1 890 27	2 047 71	925 46
June 5	3·359 770	3·463 371	1·569 250	1 948 32	2 008 39	910 00
21	3·448 016	3·382 824	1·536 839	2 005 49	1 967 57	893 88
July 7	3·534 290	3·300 283	1·503 520	2 061 75	1 925 24	877 09
23	3·618 452	3·215 854	1·469 327	2 117 01	1 881 47	859 64
Aug. 8	3·700 528	3·129 503	1·434 276	2 171 26	1 836 21	841 55
24	3·780 405	3·041 352	1·398 386	2 224 40	1 789 54	822 82
Sept. 9	3·858 065	2·951 379	1·361 672	2 276 43	1 741 44	803 45
25	3·933 461	2·859 676	1·324 150	2 327 25	1 691 94	783 44
Oct. 11	4·006 516	2·766 271	1·285 839	2 376 85	1 641 08	762 82
27	4·077 175	2·671 235	1·246 775	2 425 15	1 588 88	741 60
Nov. 12	4·145 416	2·574 609	1·206 967	2 472 11	1 535 36	719 77
28	4·211 182	2·476 449	1·166 439	2 517 66	1 480 55	697 36
Dec. 14	4·274 446	2·376 823	1·125 209	2 561 77	1 424 48	674 36
30	4·335 147	2·275 708	1 083 308	2 604 39	1 367 16	650 80
1915.						
Jan. 15	4·393 230	2·173 269	1·040 746	2 645 46	1 308 67	626 70
31	4·448 665	2·069 523	·997 573	2 684 94	1 249 04	602 07
Feb. 16	4·501 416	1·964 530	·953 779	2 722 79	1 188 29	576 92
Mar. 4	4·551 434	1·858 361	·909 420	2 758 94	1 126 48	551 26
20	4·598 701	1·751 043	·864 512	2 793 38	1 063 63	525 13
Apr. 5	4·643 163	1·642 682	·819 096	2 826 04	999 81	498 54
21	4·684 806	1·533 313	·773 138	2 856 90	935 05	471 48
May 7	4·723 588	1·423 007	·726 751	2 885 90	869 39	444 01
23	4·759 486	1·311 820	·679 902	2 913 03	802 90	416 13
June 8	−4·792 473	+1·199 815	+·632 620	+2 938 23	−735 60	−387 86

TABLE I.—*continued.*

Paris Noon.	Jovicentric Coordinates of Sun.			Reversed Attractions of Sun on Jupiter in Eighth Decimal Place.		
	x.	*y.*	*z.*	X.	Y.	Z.
1915.						
June 24	−4·822 517	+1·087 094	+·584 949	+2 961 48	−667 58	−359 21
July 10	4·849 594	·973 691	·536 935	2 982 73	598 87	330 24
26	4·873 684	·859 700	·488 584	3 001 99	529 54	300 95
Aug. 11	4·894 774	·745 157	·439 918	3 019 18	459 63	271 35
27	4·912 836	·630 191	·390 995	3 034 31	389 22	241 49
Sept. 12	4·927 859	·514 818	·341 845	3 047 34	318 36	211 39
28	4·939 837	·399 113	·292 470	3 058 26	247 09	181 07
Oct. 14	4·948 750	·283 203	·242 908	3 067 04	175 52	150 54
30	4·954 597	·167 097	·193 211	3 073 67	103 66	119 86
Nov. 15	4·957 366	+·050 894	·143 421	3 078 13	−31 60	89 05
Dec. 1	4·957 056	−·065 338	·093 484	3 080 42	+40 60	58 09
17	4·953 663	·181 554	+·043 504	3 080 53	112 90	−27 05
1916.						
Jan. 2	4·947 188	·297 625	−·006 491	3 078 44	185 20	+4 04
18	−4·937 635	−·413 511	−·056 482	+3 074 17	+257 42	+35 13

TABLE II.

Jovicentric Coordinates of Jupiter's Eighth Satellite to Seven Decimal Places.

Paris Noon.	*x.*	*y.*	*z.*
1910.			
May 25	+·025 7513	−·170 8829	−·045 6179
June 10	+·010 8491	−·167 0063	−·034 7227
26	−·004 2583	−·160 6432	−·023 3157
July 12	−·019 3516	−·151 5088	−·011 5151
28	−·034 1198	−·139 2792	+·000 5071
Aug. 13	−·048 1178	−·123 6021	+·012 4954
29	−·060 7087	−·104 1328	+·024 0674
Sept. 14	−·070 9948	−·080 6250	+·034 6554
30	−·077 7693	−·053 1207	+·043 4463
Oct. 16	−·079 5762	−·022 2791	+·049 3746
Nov. 1	−·075 0499	+·010 2410	+·051 2905
17	−·063 5923	+·041 7234	+·048 4034
Dec. 3	−·045 9764	+·069 1327	+·040 7943
19	−·024 1638	+·090 3277	+·029 4726
1911.			
Jan. 4	−·000 4085	+·104 6612	+·015 8651
20	+·023 4942	+·112 6371	+·001 2630
Feb. 5	+·046 4069	+·115 2670	−·013 4082

TABLE II.—*continued.*

Paris Noon.		*x.*	*y.*	*z.*
1911.				
Feb.	21	+ ·067 7203	+ ·113 6276	− ·027 5690
Mar.	9	+ ·087 1628	+ ·108 6698	− ·040 8872
	25	+ ·104 6544	+ ·101 1721	·053 1880
Apr.	10	+ ·120 2151	+ ·091 7522	− ·064 3908
	26	+ ·133 9128	+ ·080 8954	− ·074 4698
May	12	+ ·145 8339	+ ·068 9828	− ·083 4303
	28	+ ·156 0684	+ ·056 3154	− ·091 2944
June	13	+ ·164 7021	+ ·043 1331	− ·098 0929
	29	+ ·171 8128	+ ·029 6288	·103 8604
July	15	+ ·177 4691	+ ·015 9596	− ·108 6324
	31	+ ·181 7303	+ ·002 2552	− ·112 4438
Aug.	16	+ ·184 6466	− ·011 3759	− ·115 3277
Sept.	1	+ ·186 2601	− ·024 8412	− ·117 3153
	17	+ ·186 6056	− ·038 0601	− ·118 4354
Oct.	3	+ ·185 7114	− ·050 9607	·118 7143
	19	+ ·183 6003	− ·063 4770	− ·118 1760
Nov.	4	+ ·180 2903	− ·075 5462	− ·116 8421
	20	+ ·175 7954	− ·087 1066	− ·114 7318
Dec.	6	+ ·170 1265	− ·098 0955	− ·111 8620
	22	+ ·163 2920	− ·108 4469	− ·108 2475
1912.				
Jan.	7	+ ·155 2988	− ·118 0893	− ·103 9011
	23	+ ·146 1532	− ·126 9432	− ·098 8339
Feb.	8	+ ·135 8621	− ·134 9181	− ·093 0553
	24	+ ·124 4344	− ·141 9094	·086 5738
Mar.	11	+ ·111 8834	− ·147 7944	·079 3976
	27	+ ·098 2294	− ·152 4276	·071 5355
Apr.	12	+ ·083 5041	− ·155 6351	− ·062 9989
	28	+ ·067 7568	− ·157 2076	− ·053 8046
May	14	+ ·051 0637	− ·156 8924	− ·043 9791
	30	+ ·033 5420	− ·154 3845	− ·033 5655
June	15	+ ·015 3715	− ·149 3173	·022 6345
July	1	− ·003 1730	− ·141 2564	− ·011 3018
	17	− ·021 6728	− ·129 7029	+ ·000 2446
Aug.	2	− ·039 4957	− ·114 1209	+ ·011 7035
	18	− ·055 7073	− ·094 0206	+ ·022 6041
Sept.	3	− ·068 9898	− ·069 1459	+ ·032 2455
	19	− ·077 6631	− ·039 8152	+ ·039 6783
Oct.	5	− ·079 9833	− ·007 3529	+ ·043 8325

TABLE II.—*continued.*

Paris Noon.		*x.*	*y.*	*z.*
1912.				
Oct.	21	− ·074 8349	+ ·025 7460	+ ·043 8871
Nov.	6	− ·062 4640	+ ·056 4174	+ ·039 7283
	22	− ·044 4930	+ ·082 2081	+ ·032 0579
Dec.	8	− ·023 1226	+ ·102 0172	+ ·022 0129
	24	− ·000 3061	+ ·115 9271	+ ·010 6909
1913.				
Jan.	9	+ ·022 5790	+ ·124 6353	− ·001 0805
	25	+ ·044 6839	+ ·129 0045	− ·012 7567
Feb.	10	+ ·065 5309	+ ·129 8420	− ·024 0060
	26	+ ·084 8695	+ ·127 8266	− ·034 6369
Mar.	14	+ ·102 5832	+ ·123 5038	− ·044 5448
	30	+ ·118 6301	+ ·117 3049	− ·053 6772
Apr.	15	+ ·133 0088	+ ·109 5694	− ·062 0124
May	1	+ ·145 7388	+ ·100 5660	− ·069 5465
	17	+ ·156 8493	+ ·090 5096	− ·076 2855
June	2	+ ·166 3733	+ ·079 5746	− ·082 2405
	18	+ ·174 3441	+ ·067 9052	− ·087 4246
July	4	+ ·180 7934	+ ·055 6233	− ·091 8513
	20	+ ·185 7506	+ ·042 8348	− ·095 5334
Aug.	5	+ ·189 2425	+ ·029 6345	− ·098 4821
	21	+ ·191 2932	+ ·016 1104	− ·100 7068
Sept.	6	+ ·191 9243	+ ·002 3467	− ·102 2148
	22	+ ·191 1555	− ·011 5730	− ·103 0113
Oct.	8	+ ·189 0048	− ·025 5634	− ·103 0994
	24	+ ·185 4893	− ·039 5353	− ·102 4804
Nov.	9	+ ·180 6259	− ·053 3934	− ·101 1539
	25	+ ·174 4323	− ·067 0347	− ·099 1182
Dec.	11	+ ·166 9282	− ·080 3466	− ·096 3710
	27	+ ·158 1366	− ·093 2053	− ·092 9100
1914.				
Jan.	12	+ ·148 0857	− ·105 4746	− ·088 7338
	28	+ ·136 8108	− ·117 0047	− ·083 8431
Feb.	13	+ ·124 3566	− ·127 6312	− ·078 2417
Mar.	1	+ ·110 7800	− ·137 1747	− ·071 9385
	17	+ ·096 1532	− ·145 4410	− ·064 9489
Apr.	2	+ ·080 5671	− ·152 2212	− ·057 2974
	18	+ ·064 1355	− ·157 2927	− ·049 0196
May	4	+ ·046 9994	− ·160 4211	− ·040 1653
	20	+ ·029 3324	− ·161 3631	− ·030 8019
June	5	+ ·011 3471	− ·159 8701	− ·021 0184

TABLE II.—*continued.*

Paris Noon.		*x.*	*y.*	*z.*
1914.				
June	21	− ·006 6977	− ·155 6944	− ·010 9300
July	7	− ·024 4877	− ·148 5980	− ·000 6844
	23	− ·041 6426	− ·138 3662	+ ·009 5310
Aug.	8	− ·057 7046	− ·124 8292	+ ·019 4808
	24	− ·072 1281	− ·107 8948	+ ·028 8728
Sept.	9	− ·084 2769	− ·087 5979	+ ·037 3517
	25	− ·093 4379	− ·064 1680	+ ·044 5013
Oct.	11	− ·098 8710	− ·038 1098	+ ·049 8639
	27	− ·099 9105	− ·010 2724	+ ·052 9900
Nov.	12	− ·096 1182	+ ·018 1437	+ ·053 5224
	28	− ·087 4468	+ ·045 6971	+ ·051 2964
Dec.	14	− ·074 3254	+ ·070 9407	+ ·046 4088
	30	− ·057 5951	+ ·092 6922	+ ·039 2083
1915.				
Jan.	15	− ·038 3181	+ ·110 2093	+ ·030 2082
	31	− ·017 5638	+ ·123 2042	+ ·019 9709
Feb.	16	+ ·003 7385	+ ·131 7426	+ ·009 0180
Mar.	4	+ ·024 8567	+ ·136 1094	− ·002 2142
	20	+ ·045 2515	+ ·136 6973	− ·013 3829
Apr.	5	+ ·064 5436	+ ·133 9335	− ·024 2283
	21	+ ·082 4759	+ ·128 2400	− ·034 5573
May	7	+ ·098 8804	+ ·120 0153	− ·044 2276
	23	+ ·113 6531	+ ·109 6288	− ·053 1350
June	8	+ ·126 7357	+ ·097 4205	− ·061 2039
	24	+ ·138 1028	+ ·083 7033	− ·068 3805
July	10	+ ·147 7531	+ ·068 7662	− ·074 6280
	26	+ ·155 7033	+ ·052 8771	− ·079 9233
Aug.	11	+ ·161 9839	+ ·036 2852	− ·084 2546
	27	+ ·166 6359	+ ·019 2226	− ·087 6197
Sept.	12	+ ·169 7083	+ ·001 9056	− ·090 0246
	28	+ ·171 2561	− ·015 4645	− ·091 4824
Oct.	14	+ ·171 3385	− ·032 7004	− ·092 0123
	30	+ ·170 0174	− ·049 6283	− ·091 6387
Nov.	15	+ ·167 3557	− ·066 0871	− ·090 3903
Dec.	1	+ ·163 4162	− ·081 9273	− ·088 2993
	17	+ ·158 2605	− ·097 0101	− ·085 4009
1916.				
Jan.	2	+ ·151 9484	− ·111 2061	− ·081 7327
	18	+ ·144 5372	− ·124 3937	− ·077 3343
Feb.	3	+ ·136 0815	− ·136 4578	− ·072 2470

Ephemeris for Jupiter VIII, 1911 to 1913. Paris Noon.

Date	J. VIII – ☿. R.A.	Dec.
1911.	m s	′ ″
Feb. 5	−2 31·61	−27 13·3
9	−2 22·03	−30 43·4
13	−2 11·97	−34 13·7
17	−2 1·51	−37 43·5
21	−1 50·70	−41 12·4
25	−1 39·66	−44 39·8
Mar. 1	−1 28·40	−48 5·2
5	−1 16·99	−51 28·0
9	−1 5·48	−54 47·6
13	−0 53·96	−58 3·4
17	−0 42·44	−61 15·1
21	−0 30·96	−64 21·8
25	−0 19·56	−67 23·2
29	−0 8·35	−70 18·6
Apr. 2	+0 2·72	−73 7·4
6	+0 13·60	−75 49·2
10	+0 24·31	−78 23·4
14	+0 34·73	−80 49·1
18	+0 44·95	−83 6·3
22	+0 54·95	−85 14·5
26	+1 4·74	−87 13·4
30	+1 14·28	−89 1·4
May 4	+1 23·63	−90 39·8
8	+1 32·80	−92 8·3
12	+1 41·81	−93 27·0
16	+1 50·67	−94 35·0
20	+1 59·42	−95 33·6
24	+2 8·07	−96 22·8
28	+2 16·63	−97 3·4
June 1	+2 25·15	−97 34·6
5	+2 33·61	−97 58·1
9	+2 42·05	−98 14·0
13	+2 50·45	−98 22·9
17	+2 58·86	−98 25·7
21	+3 7·26	−98 22·4
25	+3 15·66	−98 13·8
29	+3 24·04	−98 0·0
July 3	+3 32·42	−97 42·0
7	+3 40·79	−97 19·6
11	+3 49·16	−96 53·3
15	+3 57·52	−96 23·3

Date	J. VIII – ☿. R.A.	Dec.
1912.	m s	′ ″
Mar. 11	+6 58·81	−20 50·9
15	+6 54·44	−19 14·1
19	+6 49·89	−17 37·0
23	+6 45·17	−15 59·3
27	+6 40·27	−14 21·1
31	+6 35·19	−12 42·3
Apr. 4	+6 29·91	−11 2·9
8	+6 24·42	−9 22·8
12	+6 18·71	−7 42·1
16	+6 12·81	−6 0·5
20	+6 6·64	−4 18·3
24	+6 0·20	−2 35·4
28	+5 53·45	−0 51·7
May 2	+5 46·38	+0 52·7
6	+5 38·94	+2 37·7
10	+5 31·11	+4 23·3
14	+5 22·84	+6 9·2
18	+5 14·08	+7 55·6
22	+5 4·83	+9 42·2
26	+4 55·06	+11 28·7
30	+4 44·74	+13 15·0
June 3	+4 33·78	+15 0·9
7	+4 22·25	+16 46·2
11	+4 10·12	+18 30·7
15	+3 57·41	+20 14·1
19	+3 44·01	+21 56·7
23	+3 30·05	+23 37·6
27	+3 15·52	+25 16·4
July 1	+3 0·48	+26 52·7
5	+2 44·86	+28 26·1
9	+2 28·79	+29 56·1
13	+2 12·29	+31 22·4
17	+1 55·44	+32 44·4
21	+1 38·20	+34 2·0
25	+1 20·72	+35 14·4
29	+1 3·05	+36 21·2
Aug. 2	+0 45·25	+37 21·6
6	+0 27·34	+38 15·6
10	+0 9·46	+39 2·1
14	−0 8·32	+39 40·6
18	−0 25·91	+40 10·6

Date	J VIII – ☿. R.A.	Dec.
1913.	m s	′ ″
Apr. 15	+ 8 5·71	−55 10·9
19	+ 8 19·87	−56 4·4
23	+ 8 33·71	−56 58·3
27	+ 8 47·21	−57 52·6
May 1	+ 9 0·35	−58 47·2
5	+ 9 13·15	−59 42·7
9	+ 9 25·54	−60 38·3
13	+ 9 37·50	−61 33·9
17	+ 9 49·00	−62 29·0
21	+10 0·04	−63 24·1
25	+10 10·55	−64 18·2
29	+10 20·48	−65 10·9
June 2	+10 29·79	−66 1·8
6	+10 38·44	−66 50·8
10	+10 46·38	−67 36·9
14	+10 53·56	−68 19·7
18	+10 59·96	−68 58·6
22	+11 5·46	−69 33·1
26	+11 10·10	−70 2·6
30	+11 13·91	−70 26·5
July 4	+11 16·69	−70 44·6
8	+11 18·50	−70 55·7
12	+11 19·40	−71 0·1
16	+11 19·35	−70 57·2
20	+11 18·40	−70 47·0
24	+11 16·44	−70 27·8
28	+11 13·62	−70 1·0
Aug. 1	+11 9·97	−69 26·6
5	+11 5·52	−68 44·9
9	+11 0·29	−67 55·1
13	+10 54·36	−66 58·4
17	+10 47·77	−65 55·1
21	+10 40·56	−64 45·6
25	+10 32·75	−63 29·7
29	+10 24·44	−62 8·5
Sept. 2	+10 15·67	−60 42·4
6	+10 6·52	−59 12·1
10	+ 9 56·68	−57 37·3
14	+ 9 46·60	−55 59·7
18	+ 9 36·32	−54 19·6
22	+ 9 25·94	−52 38·1

Ephemeris for Jupiter VIII, 1914–1915. *Paris Noon.*

Date.	J. VIII—$\mathcal{Y}$. R.A.	Dec.
1914.	m s	′ ″
May 20	−5 24·66	+ 0 52·2
24	−5 37·81	+ 1 40·9
28	−5 50·51	+ 2 33·1
June 1	−6 2·74	+ 3 28·7
5	−6 14·48	+ 4 27·7
9	−6 25·76	+ 5 31·3
13	−6 36·47	+ 6 38·1
17	−6 46·62	+ 7 48·4
21	−6 56·14	+ 9 1·4
25	−7 5·04	+10 18·2
29	−7 13·23	+11 37·1
July 3	−7 20·69	+12 58·0
7	−7 27·35	+14 20·0
11	−7 35·15	+15 43·5
15	−7 38·08	+17 7·0
19	−7 42·09	+18 29·8
23	−7 45·18	+19 51·1
27	−7 47·23	+21 10·3
31	−7 48·30	+22 26·4
Aug. 4	−7 48·35	+23 38·6
8	−7 47·37	+24 46·3
12	−7 45·28	+25 47·9
16	−7 42·15	+26 43·5
20	−7 37·96	+27 32·5
24	−7 32·72	+28 14·5
28	−7 26·41	+28 47·7
Sept. 1	−7 19·06	+29 13·3
5	−7 10·69	+29 30·8
9	−7 1·31	+29 40·5
13	−6 50·91	+29 40·8
17	−6 39·54	+29 33·5
21	−6 27·20	+29 18·6
25	−6 13·93	+28 56·6
29	−5 59·72	+28 27·1
Oct. 3	−5 44·63	+27 51·4
7	−5 28·68	+27 10·0
11	−5 11·90	+26 23·6
15	−4 54·30	+25 32·1
19	−4 35·96	+24 37·1
23	−4 16·90	+23 39·0
27	−3 57·18	+22 38·8

Date.	J. VIII—$\mathcal{Y}$. R.A.	Dec.
1915.	m s	′ ″
June 24	+4 5·86	−44 1·6
28	+3 56·96	−45 47·9
July 2	+3 47·83	−47 33·1
6	+3 38·50	−49 16·6
10	+3 29·00	−50 57·8
14	+3 19·39	−52 35·7
18	+3 9·67	−54 10·2
22	+2 59·88	−55 40·5
26	+2 50·03	−57 6·2
30	+2 40·16	−58 26·4
Aug. 3	+2 30·28	−59 41·2
7	+2 20·41	−60 49 6
11	+2 10·56	−61 51·5
15	+2 0·79	−62 45·6
19	+1 51·06	−63 32·3
23	+1 41·38	−64 11·0
27	+1 31·75	−64 42·0
31	+1 22·21	−65 3·3
Sept. 4	+1 12·70	−65 16·5
8	+1 3·22	−65 21·6
12	+0 53·75	−65 18·9
16	+0 44·32	−65 7·4
20	+0 34·87	−64 49·0
24	+0 25·38	−64 23·7
28	+0 15·83	−63 52·3
Oct. 2	+0 6·19	−63 14·8
6	−0 3·54	−62 32·4
10	−0 13·39	−61 45·8
14	−0 23·34	−60 55·5
18	−0 33·47	−60 2·8
22	−0 43·72	−59 7·7
26	−0 54·45	−58 11·3
30	−1 4·64	−57 13·8
Nov. 3	−1 15·33	−56 16·7
7	−1 26·13	−55 19·7
11	−1 37·04	−54 23·3
15	−1 48·03	−53 27·6
19	−1 59·13	−52 34·1
23	−2 10·27	−51 41·9
27	−2 21·45	−50 51·3
Dec. 1	−2 32·64	−50 2·4

The more striking features of the orbit are exhibited in the accompanying diagram, in which it is projected on a plane from which it does not depart far during the year 1908. The divergence afterwards becomes sensible, and by the middle of 1910 the angular distance from this plane is fully 5° and increases with each revolution, and by the beginning of 1916 is nearly 15°. This is represented in the diagram by means of the dotted arrows, the lengths of which indicate the amount the orbit is out of the plane of projection and the direction up or down, whether it is above or below this plane. The double-headed arrows show the points where the orbit crosses the plane.

The dotted arcs show those parts of the orbit actually observed in 1908, 1909, and 1910. The larger dots indicate the dates of observation.

The orbits of the other satellites are inserted for comparison. They are drawn to scale, but unforeshortened.

The long arrows indicate the approximate direction of the Earth at the date of opposition in each year.

The points 0^h, 6^h, 12^h, 18^h are Jovicentric R.A.

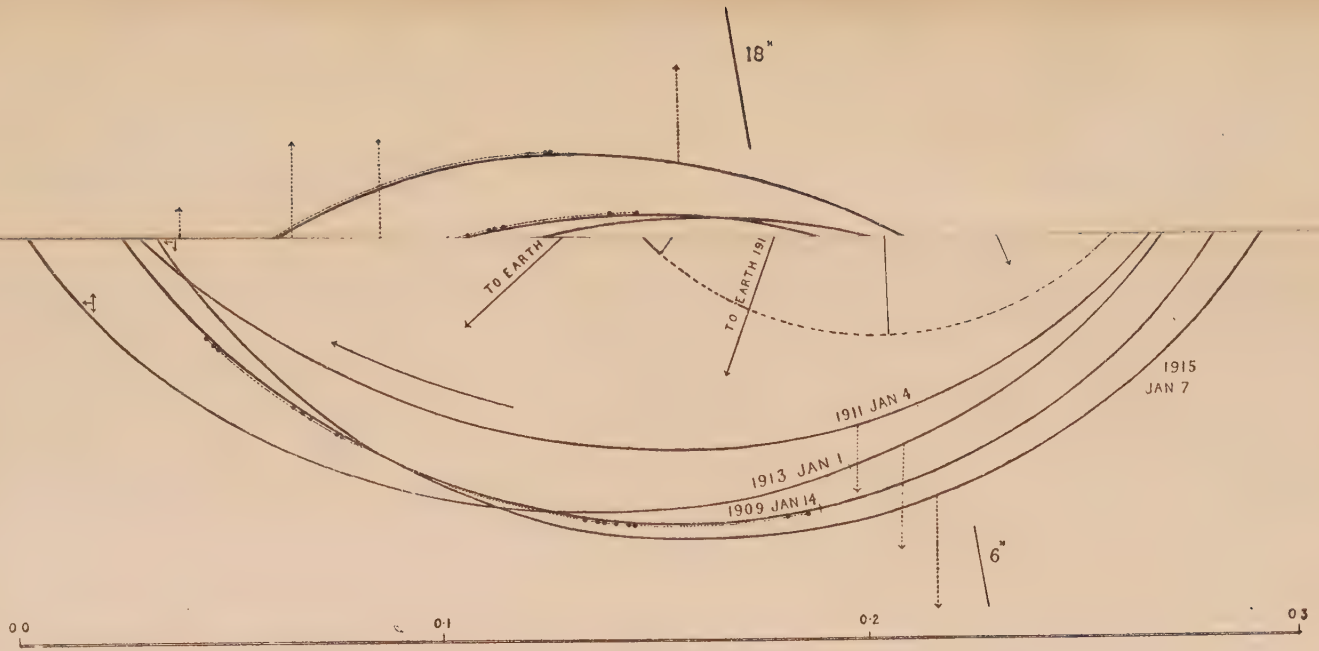
The features of the orbit can be better exhibited in a model than in a diagram. Such a model has been constructed under the direction of Mr. Davidson, showing the path of the eighth satellite from 1910 to 1916, with the orbits of the other satellites in their proper planes, on the scale 80 inches = 1 solar unit. This was exhibited to the Society.

Observations of Jupiter's Eighth Satellite in 1910.

(*Communicated by the Astronomer Royal.*)

The eighth satellite of Jupiter was under observation near the opposition of 1910 March, and photographs were obtained with the 30-inch reflector whenever possible. Owing to the low altitude of Jupiter and the unfavourable weather, successful photographs were only secured on five nights between January 19 and April 28.

The position of the satellite has been measured on each photograph with reference to four comparison stars of the twelfth to thirteenth magnitude, symmetrically distributed about the satellite. The positions of these comparison stars have been determined on photographs of 20^m exposure, taken for the purpose with the astrographic 13-inch refractor. By these means the position the satellite would have occupied had it been possible to photograph it on the astrographic plates is determined. Its



ORBIT OF JUPITER'S EIGHTH SATELLITE, 1908-1916.

right ascension and declination are then deduced in the usual way by reference to the stars on the astrographic plate whose positions are given in the catalogues of the Astronomische Gesellschaft.

As Jupiter moves slowly, one astrographic reference plate served for several photographs of the satellite.

To determine the position of the satellite relatively to Jupiter, the right ascensions and declinations of the planet given in the *Nautical Almanac* have been corrected, as shown in the latter part of this paper. A comparison has been made with the Ephemeris published in *Monthly Notices* for 1909 March.

Observations of Satellite VIII.

Date and G.M.T.					Exposure.	Apparent R.A.			Apparent Dec.			Sat. VIII.—Jupiter.			Tab. − Obs.	
												R.A.		Dec.	R.A.	Dec.
1910.	d	h	m	s	m	h	m	s	°	′	″	m	s	″	s	″
Jan.	19	15	9	11	110	13	2	38·55	−5	41	14·1	+7	27·30	−77 12·0	+0·12	+ 5·4
Feb.	11	15	5	9	120	13	3	12·80	5	31	2·5	8	22·73	75 42·3	+0·11	+ 6·9
	11	17	4	19	100	13	3	12·30	5	30	57·5	8	22·98	75 43·3	+0·03	+ 8·5
	15	14	1	28	100	13	2	40·17	5	25	10·4	8	30·95	75 8·6	−0·06	+ 6·1
	15	15	52	9	100	13	2	39·28	5	25	2·9	8	30·96	75 8·0	+0·08	+ 6·2
Apr.	27	9	43	31	90	12	34	54·43	1	54	29·7	8	57·54	48 18·6	−0·34	+10·4
	27	11	7	8	60	12	34	53·04	1	54	18·7	8	57·40	48 15·2	−0·28	+ 9·0
	28	9	56	22	90	12	34	31·53	1	51	46·3	8	56·20	47 43·7	−0·31	+ 9·3
	28	11	33	6	90	12	34	30·03	1	51	36·0	8	56·12	−47 41·8	−0·32	+ 9·7

Jan. 19. Very faint.
Feb. 11. Second photograph. Very diffused and indefinite.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

Errors of the Tabular Place of Jupiter.—To eliminate the error of the tabular place of Jupiter (for comparison with the R.A. and Dec. of the satellite deduced as above), and also that arising from any systematic error of the catalogues employed, a series of photographs of Jupiter was taken with the 26-inch refractor, using the occulting shutter. By this means good measurable images of Jupiter were obtained, together with sufficiently exposed images of the reference stars. Four images of Jupiter and of four or six stars were measured on each plate, the positions of the stars being deduced from measures made on the astrographic reference plates. The observed positions of Jupiter are thus affected by any systematic errors of the catalogue, and are comparable, therefore, with the positions of the satellites deduced in the same manner. Seven photographs were selected for measurement, and the following are the results obtained:—

Errors of Tabular Place of Jupiter.

1910.	Tab. − Obs. R.A.	Dec.
	s	″
April 5	−0·05	0·0
5	−0·05	+0·1
25	0·00	...
25	+0·01	+0·5
27	−0·01	+0·2
28	0·00	+0·4
28	+0·01	+0·4
Mean	−0·013	+0·27

The photographs of Jupiter were all taken in April when Jupiter and the satellite were in the region covered by the A.G. (Nicolajew) Catalogue.

When photographed in January and February the satellite was in the region covered by the A.G. (Strasburg) Catalogue. From 25 stars common to the two catalogues between R.A. 11½ hours and 13 hours, the correction Strasburg − Nicolajew was found to be

R.A. $+0^{s}\cdot016$ Dec. $-0''\cdot20$.

The following corrections have therefore been applied to Jupiter's tabular place :—

		s	″
January and February	R.A.	+0·03	Dec. −0·5
April		+0·01	−0·3

Royal Observatory, Greenwich:
1910 *November* 4.

Observations of Minor Planets in 1909.

(*Communicated by the Astronomer Royal.*)

The following positions were obtained from photographs taken with the 30-inch reflector and the 13-inch refractor at the Royal Observatory, Greenwich. The plates were measured with the astrographic micrometer. Six reference stars were, as a rule, measured with the planets, their positions being derived, when possible, from the catalogues of the Astronomische Gesellschaft, or from the Radcliffe Catalogue 1890. As the mean places of the surrounding stars were used, the positions given are referred to the equator and equinox of 1909·0, and are automatically corrected for the part of the aberration arising from the Earth's motion. They are not corrected for parallax or the part of the aberration arising from the planet's motion.

The mean places for 1909·0 may be obtained by applying the corrections for parallax and Earth's aberration, and antedating for the light-time.

Log Parallax Correction = log Parallax − log Δ.

Date and G.M.T. 1909.					R.A. 1909·0.			Log Parallax Factor.	Dec. 1909·0.			Log Parallax Factor.	Correction for Earth's Aberration.	
	d	h	m	s	h	m	s		°	′	″		R.A. s	Dec. ″
								1909. FN.						
Jan.	16	13	14	40	10	55	7·28	−9·257	+ 6	45	21·2	+0·788	+0·88	−5·4
	16	15	3	7	10	55	6·22	−8·110		6	45	39·7	0 789	0·88	5·4
	19	13	31	47	10	54	19·37	−9·136		6	58	18·7	0·791	0·93	5·7
	25	13	27	0	10	52	13·88	−9·021		7	27	42·7	0·786	1·03	6·4
Feb.	12	11	35	15	10	42	34·42	−9·187		9	18	49·3	0·775	1·24	7·5
	13	11	29	23	10	41	55·43	−9·191		9	25	44·3	0·774	1·24	7·6
	19	11	6	4	10	37	49·62	−9·176		10	8	3·8	0·768	1·27	7·6
								1909. HB.						
Apr.	7	8	30	39	10	22	56·36	−8·914	+10	29	36·5	+0·760	+0·94	−5·3
	8	9	10	6	10	22	31·08	−8·055		10	33	2·1	0·759	0·92	5·2
	9	9	5	11	10	22	8·31	−8·055		10	36	9·5	0·767	0·90	5·1
	10	9	32	11	10	21	46·36	+8·590		10	39	12·7	0·757	0·89	4·9
	14	9	56	54	10	20	36·42	+9·018		10	49	32·5	0·758	0·81	4·5
	16	9	47	33	10	20	11·56	+9·018		10	53	38·9	0·756	0·77	4·3
	18	9	48	40	10	19	53·07	+9·073		10	57	0·4	0·758	0·74	4·1
								532. Herculina.						
May	19	11	58	49	17	26	52·45	−9·181	− 7	32	12·6	+0·872	+1·26	−3·3
	21	11	41	3	17	25	19·18	−9·225		7	36	43·2	0·872	1·28	3·2
June	22	10	37	12	16	56	8·62	−8·440		10	0	57·7	0·887	1·33	0·9

Date and G.M.T. 1909.	R.A. 1909·0.	Log Parallax Factor.	Dec. 1909·0.	Log Parallax Factor.	Correction for Earth's Aberration. R.A.	Dec.
d h m s	h m s		° ′ ″		s	″
			124. Alkeste.			
May 18 10 15 4	10 21 0·71	+9·480	+ 9 34 58·4	+0·794	+0·13	−1·0
19 10 15 35	10 21 47·68	+9·486	9 31 38·0	0·795	0·12	0·9
20 10 32 8	10 22 36·31	+9·508	9 28 8·1	0·800	0·10	0·8
21 10 40 26	10 23 25·57	+9·520	9 24 32·3	0·803	+0·08	−0·7
29 10 1 35	10 30 30·68	+9·504	8 51 50·1	0·802	−0·04	+0·1
29 10 14 11	10 30 31·14	+9·518	8 51 48·2	0·805	−0·04	+0·1
			483. Seppina.			
May 25 10 49 47	13 52 27·62	+9·038	+ 5 38 58·5	+0·800	+1·07	−2·3
			163. Erigone.			
May 25 12 19 48	15 41 11·59	+8·909	−11 56 53·8	+0·893	+1·35	−4·3
			660. 1908. CC.			
June 22 10 20 28	16 4 54·33	+8·435	+ 4 43 50·3	+0·805	+1·19	+0·9
			288. Glauke.			
June 22 10 54 52	17 23 0·89	−8·641	−18 49 48·8	+0·916	+1·42	−1·0
			527. Euryanthe.			
June 22 11 16 47	17 32 46·68	−8·362	−13 58 38·7	+0·902	+1·39	−0·5
			509. Iolanda.			
June 22 11 40 22	17 40 33·50	+7·507	− 6 11 37·8	+0·870	+1·36	−0·1
			511. Davida.			
June 22 11 59 31	18 2 7·18	−7·222	−16 30 57·8	+0·910	+1·42	+0·1
30 10 36 27	17 55 47·88	−8·883	16 52 46·2	0·910	1·41	0·3
			479. Caprera.			
June 22 12 53 45	18 31 57·52	+8·597	−14 35 23·5	+0·903	+1·40	+0·8
July 19 11 3 56	18 8 16·97	+8·861	15 47 55·5	0·906	1·29	1·4
20 11 59 54	18 7 29·96	+9·226	15 51 22·5	0·900	1·28	1·4
			46. Hestia.			
June 30 10 59 5	18 31 15·60	−8·995	−19 15 8·7	+0·915	+1·44	+1·2
July 19 11 33 22	18 13 36·76	+9·056	19 34 25·7	0·915	1·33	1·0
20 12 23 51	18 12 48·55	+9·301	19 35 40·7	0·905	1·32	0·3

Date and G.M.T. 1909.	R.A. 1909·0.	Log Parallax Factor.	Dec. 1909·0.	Log Parallax Factor.	Correction for Earth's Aberration.	
					R.A.	Dec.
d h m s	h m s		° ′ ″		s	″
437. Rhodia.						
June 30 12 0 6	19 2 35·62	−8·686	−17 44 32·2	+0·913	+1·42	+2·0
July 23 12 7 45	18 41 47·23	+9·163	−15 45 41·1	0·902	1·33	2·3
472. Roma.						
July 23 11 45 40	18 46 55·75	+9·012	−14 54 14·5	+0·902	+1·33	+2·5
Aug. 12 10 42 2	18 34 12·76	+9·171	17 19 53·8	0·906	1·06	2·3
433. Eros.						
July 23 13 31 53	3 50 28·96	−9·623	+29 32 32·2	+0·790	−0·74	−5·0
Aug. 9 13 32 56	4 50 53·34	−9·637	32 21 12·0	0·768	0·74	4·2
14 14 16 59	5 9 56·56	−9·634	32 53 15·8	0·721	0·73	4·0
16 14 5 35	5 17 37·43	−9·639	33 3 19·3	0·732	0·73	3·9
19. Fortuna.						
Aug. 7 11 20 40	19 38 44·25	+8·878	−18 50 50·7	+0·915	+1·32	+3·5
9 10 18 0	19 37 8·03	−8·131	18 55 36·7	0·917	1·30	3·5
95. Arethusa.						
Aug. 7 11 44 13	20 7 59·57	+8·804	− 2 19 12·4	+0·849	+1·29	+5·6
9 10 37 3	20 6 30·62	−8·459	2 23 19·9	0·850	1·28	5·8
313. Chaldæa.						
Aug. 7 12 11 56	21 3 45·34	+8·284	− 4 26 0·3	+0·861	+1·31	+5·8
9 12 15 43	21 1 57·58	+8·602	4 39 43·5	0·862	1·31	6·0
57. Mnemosyne.						
Aug. 7 12 32 20	22 8 40·56	−8·724	+ 7 1 4·8	+0·788	+1·24	+4·9
9 11 59 30	22 7 27·92	−8·958	6 53 23·1	0·872	1·26	5·2
429. Lotis.						
Aug. 11 11 18 9	21 2 2·18	−8·601	− 0 17 37·9	+0·838	+2·09	+6·1
12 11 12 6	21 1 11·54	−8·618	0 22 8·8	0·838	1·30	6·2
8. Flora.						
Nov. 5 10 54 25	3 50 52·13	−9·262	+ 9 9 9·0	+0·779	+1·29	+5·2
21 10 51 2	3 34 34·15	−8·830	8 55 50·0	0·773	1·34	4·6
22 9 53 39	3 33 34·58	−9·174	8 56 19·6	0·777	1·34	4·5
Dec. 1 9 58 17	3 24 53·32	−8·869	9 9 17·3	0·772	1·28	3·9
3 9 10 22	3 23 12·76	−9·129	9 14 8·7	0·774	1·27	3·8
8 10 10 48	3 19 24·30	−∞	9 29 53·3	0·767	1·22	3·5
20 7 37 41	3 13 50·01	−9·201	10 24 48·9	0·767	1 06	2·6

Date and G.M.T. 1909.	R.A. 1909·0.	Log Parallax Factor.	Dec. 1909·0.	Log Parallax Factor.	Correction for Earth's Aberration. R.A.	Dec.
d h m s	h m s		° ′ ″		s	″
		334. Chicago.				
Nov. 30 11 54 35	4 29 58·94	+7·221	+16 4 48·4	+0·704	+1·41	+3·1
Dec. 1 9 45 6	4 29 20·30	−9·293	16 3 40·5	0·723	1·41	3·1
3 9 32 56	4 27 55·41	−9·302	16 1 19·8	0·724	1·40	3·1
8 11 2 27	4 24 22·17	−8·299	15 55 57·5	0·706	1·39	3·0
		50. Virginia.				
Nov. 30 11 54 35	4 30 39·91	+7·222	+16 42 26·0	+0·698	+1·41	+3·1
Dec. 1 9 45 6	4 29 46·10	−9·294	16 40 19·2	0·717	1·41	3·1
3 9 32 56	4 27 48·83	−9·303	16 35 57·1	0·719	1·40	3·1
8 11 2 27	4 23 1·37	−8·301	16 26 13·8	0·701	1·39	3·0

Royal Observatory, Greenwich:
1910 *November* 4.

Mean Areas and Heliographic Latitudes of Sun-spots in the Years 1907, 1908, *and* 1909.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxx. p. 76, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich; at Dehra Dûn; at the Kodaikánal Observatory, India; and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in the three years 1907, 1908, and 1909; and Table II. gives the same particulars for each of the entire years 1901 to 1909 inclusive. The areas are given in two forms: first, projected areas, that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in the three years 1907 to 1909 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots; and Table IV. gives

the same information for each year as a whole for the years 1901 to 1909 inclusive. Table II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol lxiii., and for the years 1901 to 1906 on pp. 77 and 78 of vol. lxx.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. I. being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
			Projected.			Corrected for Foreshortening.		
			Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
	1906. d							
712	Dec. 15·64	27	297	1913	2072	222	1493	2334
	1907.							
713	Jan. 11·97	27	312	2040	2434	221	1465	2515
714	Feb. 8·32	28	490	3034	2416	349	2236	2600
715	Mar. 7·65	27	151	980	2250	106	710	2432
716	Apr. 3·95	27	134	882	1850	93	628	1970
717	May 1·21	28	139	886	1418	97	655	1536
718	May 28·43	27	199	1398	1303	145	1014	1419
719	June 24·63	27	163	1062	1459	119	820	1548
720	July 21·84	27	114	818	1314	95	697	1424
721	Aug. 18·06	27	201	1456	1514	141	1080	1688
722	Sept. 14·31	27	222	1493	2245	166	1169	2407
723	Oct 11·59	28	259	1708	1812	195	1263	1960
724	Nov. 7·89	27	248	1513	2083	180	1132	2191
725	Dec. 5·20	28	224	1492	2007	154	1069	2156
	1908.							
726	Jan. 1·53	27	110	708	2305	80	545	2422
727	Jan. 28·87	28	49	295	2348	36	225	2477
728	Feb. 25·21	27	57	391	1851	43	305	1973
729	Mar. 23·53	27	121	843	1860	82	587	1935
730	Apr. 19·80	27	99	703	1696	78	573	1834
731	May 17·04	28	145	1057	1685	96	713	1755
732	June 13·24	27	63	433	1665	47	319	1714
733	July 10·44	27	167	1093	2015	110	734	2093

TABLE I.—*continued.*

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	1908. d							
734	Aug. 6·65	27	354	2395	2174	258	1780	2320
735	Sept. 2·90	27	360	2210	2271	262	1614	2456
736	Sept. 30·17	28	91	573	2670	69	444	2750
737	Oct. 27·46	27	189	1123	1950	125	749	1990
738	Nov. 23·76	27	113	588	1697	88	449	1721
739	Dec. 21·09	27	205	1123	1410	151	862	1528
	1909.							
740	Jan. 17·42	28	264	1536	1495	186	1095	1561
741	Feb. 13·76	27	236	1348	1953	169	1004	1977
742	Mar. 13·09	27	223	1365	1738	152	944	1821
743	Apr. 9·39	28	72	452	1400	53	329	1427
744	May 6·64	27	150	709	1250	107	514	1271
745	June 2·85	27	70	399	948	54	307	1026
746	June 30·05	28	140	813	1280	93	546	1322
747	July 27·25	25	57	289	1149	42	225	1221
748	Aug. 23·48	28	106	537	976	96	485	1025
749	Sept. 19·74	27	303	1547	1317	221	1170	1382
750	Oct. 17·03	28	122	655	1073	100	545	1128
751	Nov. 13·33	27	240	1263	1237	164	889	1278
752	Dec. 10·64	27	235	1213	1078	167	865	1086

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1901	359	14	41	23	9	29	29
1902	349	14	86	163	10	62	178
1903	350	67	434	875	51	340	970
1904	363	93	653	1639	67	488	1761
1905	364	230	1637	2433	163	1191	2612
1906	364	158	1047	2207	114	778	2320
1907	365	221	1453	1859	160	1082	1999
1908	366	148	952	2000	106	697	2098
1909	364	171	941	1302	124	692	1354

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1906. d			°		°	°	°
712	Dec. 15·64	27	1034	11·13	459	15·85	+ 2·84	12·58
	1907.							
713	Jan. 11·97	27	832	14·81	632	14·62	+ 2·11	14·73
714	Feb. 8·32	28	555	12·24	1682	14·45	− 7·83	13·90
715	Mar. 7·65	27	266	11·10	444	15·00	− 5·21	13·54
716	Apr. 3·95	27	432	11·29	196	11·81	+ 4·08	11·45
717	May 1·21	28	171	5·84	484	11·69	− 7·12	10·17
718	28·43	27	50	10·99	963	14·58	−13·31	14·40
719	June 24·63	27	186	4·18	634	10·08	− 6·84	8·74
720	July 21·84	27	236	9·21	461	14·71	− 6·62	12·85
721	Aug. 18·06	27	603	9·12	478	15·50	− 1·77	11·94
722	Sept. 14·31	27	656	7·84	513	13·82	− 1·67	10·47
723	Oct. 11·59	28	743	7·88	520	16·64	− 2·21	11·49
724	Nov. 7·89	27	937	10·91	195	11·91	+ 6·98	11·08
725	Dec. 5·20	28	504	10·49	565	10·28	− 0·49	10·38
	1908.							
726	Jan. 1·53	27	216	11·14	329	7·83	− 0·31	9·14
727	28·87	28	128	16·98	97	12·05	+ 4·45	14·85
728	Feb. 25·21	27	41	9·61	264	8·16	− 5·77	8·35
729	Mar. 23·53	27	34	10·92	553	12·05	−10·71	11·99
730	Apr. 19·80	27	126	11·24	446	11·31	− 6·34	11·30
731	May 17·04	28	166	10·80	547	8·14	− 3·73	8·76
732	June 13·24	27	87	10·08	232	12·88	− 6·64	12·12
733	July 10·44	27	263	11·53	471	14·51	− 5·17	13·44
734	Aug. 6·65	27	785	8·20	995	10·89	− 2·47	9·70
735	Sept. 2·90	27	993	9·97	621	7·42	+ 3·28	8·99
736	30·17	28	220	10·36	224	8·31	+ 0·94	9·32
737	Oct. 27·46	27	659	11·27	90	11·48	+ 8·55	11·30
738	Nov. 23·76	27	352	11·62	96	13·51	+ 6·22	12·02
739	Dec. 21·09	27	637	9·69	226	9·95	+ 4·55	9·76
	1909.							
740	Jan. 17·42	28	615	8·61	480	10·21	+ 0·36	9·31
741	Feb. 13·76	27	412	8·53	592	10·26	− 2·55	9·55
742	Mar. 13·09	27	587	10·43	357	11·35	+ 2·20	10·77
743	Apr. 9·39	28	110	8·22	220	9·52	− 3·61	9·09
744	May 6·64	27	63	9·92	451	16·19	−13·01	15·42

TABLE III.—*continued.*

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
	1909. d			°		°	°	°
745	June 2·85	27	29	6·98	278	8·86	− 7·36	8·68
746	30·05	28	479	7·67	68	4·15	− 6·20	7·23
747	July 27·25	25	140	9·17	85	15·34	− 0·09	11·50
748	Aug. 23·48	28	37	12·36	448	6·25	− 4·84	6·71
749	Sept. 19·74	27	116	9·80	1054	6·63	− 5·00	6·95
750	Oct. 17·03	28	200	6·16	346	9·29	− 3·63	8·14
751	Nov. 13·33	27	402	12·71	488	11·41	− 0·52	12·00
752	Dec. 10·64	27	443	9·81	422	14·12	− 1·86	11·91

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heliographic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heliographic Latitude.	Mean of Daily Areas.	Mean Heliographic Latitude.		
			°		°	°	°
1901	359	22	8·59	7	16·27	+2·82	10·37
1902	349	42	18·81	21	15·29	+7·48	17·64
1903	350	132	18·11	208	21·15	−5·85	19·94
1904	363	268	16·33	220	16·88	+1·37	16·57
1905	364	750	11·66	440	15·55	+1·60	13·10
1906	364	539	13·98	239	14·01	+5·38	13·99
1907	365	488	10·12	593	13·77	−2·98	12·12
1908	366	316	10·42	381	10·34	−0·92	10·38
1909	364	299	9·45	393	9·92	−1·55	9·71

The principal features of the record for the three years 1907, 1908, and 1909 are—

1. A great increase in the mean daily spotted area for 1907 as compared with 1906; indeed the level of 1905 was almost reached, the umbræ showing 98 per cent., the whole spots 91 per cent. of the areas shown in that year. So strongly marked a revival following a decrease so considerable as that shown by 1906 is quite exceptional; the curve at maximum of the present cycle shows therefore two well-defined peaks, separated by a deep depression. In 1908 the mean daily spotted area dropped a little below the level of 1906, but this downward tendency was checked in 1909, which gave an area for whole spots about equal to that of 1908, and for umbræ a

slight increase. From this check in the decline, following on the remarkable second maximum given by 1907, the apex of the final smoothed curve will probably fall early in 1907, or thirteen years and a half later than the apex of the curve in the preceding cycle. The period, therefore, on this occasion exceeds the average by more than two years.

2. An examination of the results for the separate rotations shows that the highest value during the entire maximum was obtained for the mean daily spotted area in rotation 714, commencing 1907 February 8. This closed a short period of special activity that had prevailed during the winter of 1906–7. For the greater part of 1907 a fair amount of activity was maintained, rising to a higher level during the autumn, and diminishing sharply during the winter. The following year, 1908, opened quietly, but a striking though short-lived revival marked the months of August and September, rotation 734 giving a mean area exceeded only four times during the present maximum, *i.e.* three times in 1905 and once in 1907. A decline followed during the autumn, succeeded by a moderate revival in the winter. In 1909, as in 1907, the spring and summer were relatively quiet, and were followed by a revival of activity as the year advanced; but though the mean daily area for the year as a whole was practically the same for 1909 as for 1908, the facts that the fluctuations in activity were not quite so great, nor the periods of the minor undulations of the curve quite so long, indicate that the minimum phase of the curve is approaching.

3. The curve shown by the areas of the faculæ displayed some marked differences from that given by the sun-spots. First, of all, it has been, as usual, the smoother of the two curves; and next, its crest was reached in 1905, and its downward progress since that date has been, on the whole, fairly regular. Third, the check in this decline was felt by the faculæ a full year earlier than with the spots; for whilst with the spots 1908 and 1909 were nearly equal in area, and were both much inferior to 1907, with the faculæ 1908 rose above the level of 1907, but 1909 fell far below 1908.

4. Comparing the whole spots of the two hemispheres, the southern hemisphere has tended in general to be slightly but distinctly more prolific than the northern during each of the three years 1907, 1908, and 1909. The oscillations in preponderance from the one hemisphere to the other have been frequent, but on the whole the northern spotted area for 1907 and 1908 has been to the southern as 45 to 55; for 1909 it has been as 43 to 57. In the two preceding cycles the balance had turned heavily in favour of the southern hemisphere by the time that the maximum had been reached, and it remained generally in the same sense during the whole period of the decline.

5. The mean distance from the equator of all spots has shown a steady decrease from 1906 to the end of 1909, and would appear to indicate that about three more years will bring the period of minimum.

6. The Sun was free from spots on one day in 1907, on six days in 1908, and on five in 1909, as against five days in 1906 and two in 1905.

7. The distribution of spots in latitude during the three years has been very characteristic of the declining phase of the cycle. In the northern hemisphere no spots were seen in higher latitudes than $+27^\circ$ in 1907 and 1908, or than $+18^\circ$ in 1909. The decline in the southern hemisphere has not been quite so pronounced, for spots attained a latitude of -29° in 1907, of -25° in 1908, and -24° in 1909. In both hemispheres the disturbed zones extended downwards to the equator.

8. The number of separate groups of spots diminished steadily during the three years. In 1906 it was 351; in 1907, 329; in 1908, 304; and in 1909, 237. As the mean daily spotted area for 1909 was practically the same as in 1908, the falling off in the number of separate groups involved an increase in the average size of the group. This was due to a marked diminution in the number recorded of very small and very short-lived groups.

9. Of the 329 separate groups in 1907, 151 were in the northern hemisphere and 178 in the southern. The 304 groups of 1908 were divided into 129 north and 175 south, and the 237 of 1909 into 95 north and 142 south. Expressed in percentages, the proportion in 1907 was 46 north to 54 south; in 1908, 42 north to 58 south; and in 1909, 40 north to 60 south,—the preponderance of the southern hemisphere therefore steadily increasing as time went on.

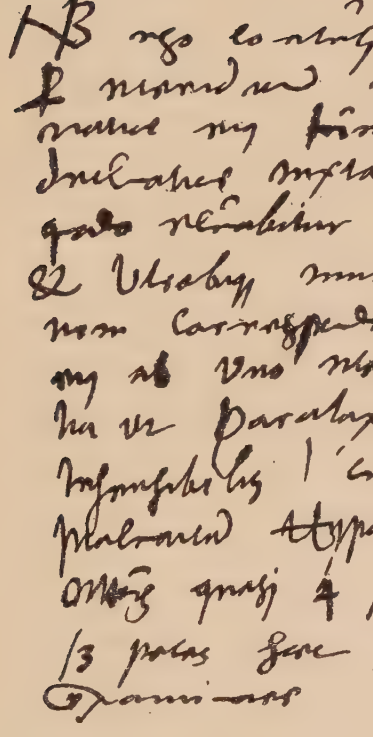
Royal Observatory, Greenwich:
1910 *November* 3.

Note on Tycho Brahe's opinion about the Solar Parallax.

By J. L. E. Dreyer. (Plate 4.)

From the time of Ptolemy to that of Kepler no serious attempt was made to improve on the value of the solar parallax adopted by Ptolemy, viz. $2'\,51''$. Mercury and Venus, being supposed to move in orbits smaller than the solar orbit, ought to have exhibited parallaxes well within the range of the instruments of the Middle Ages, especially Mercury, which at inferior conjunction ought to have had a parallax equal to that of the Moon at apogee, or $54'$. It may possibly have been this circumstance which induced Ibn Iûnis to reduce the solar parallax from $3'$ to $2'$, or rather to $1'\,57''$, though he does not give any reason for doing so. Geber blamed Ptolemy for having said that the parallaxes of the planets are insensible, and remarked that in that case he ought to have placed Venus and Mercury above the Sun.* Copernicus made no

* Tycho ought to have known this remark of Geber's, as it occurs in his book, translated by Gherardo of Cremona, and published at Nürnberg in 1534 with Apian's *Instrumentum primi mobilis* (introd., p. 3, and lib. vii. p. 104).



NOTE BY TYCHO BRAHE ON THE PARALLAX OF VENUS, 1590.

attempt to redetermine the important constant; he put the mean parallax = 3′ 1″, and the mean distance 1142 semidiameters of the Earth. But it is more remarkable that Tycho never attacked this problem, though he otherwise did not take anything for granted on the authority of the ancients. Throughout his life he used the old value. Thus, at the beginning of his observations made in the year 1578, he gives a table of parallax in altitude for every five degrees, beginning with 2′ 58″. In reducing his observations of the Sun in his MS. observations, as well as in his *Progymnasmata*, he always used the old value, although he was thereby obliged to assume that refraction acted differently on the Sun and on stars, and to construct separate refraction tables for Sun and stars.*

But it is very interesting to see that Tycho was quite aware of the very insecure foundation on which the assumed value of the solar parallax rested. In the beginning of 1590, when Venus was visible in the evening, the planet was observed as often as possible with every care, frequently by Tycho himself, and from the 28th February as early as 1 o'clock in the afternoon. Under the 3rd March there is written in Tycho's own hand :—

"*N.B.*—Ego contuli transitum ♀ per meridianum vna cum declinatione eius tunc cum declinatione iuxta 90 gradum quando eleuabatur quasi partibus 20 & vtrobique inveni ♀ declinationem correspondere mutationi eius ab vno meridie in alterum, ita vt parallaxis sit prorsus insensibilis. Cum tamen iuxta Ptolemaicam Hypothesin debeat esse minutorum quasi 4, satis perceptibiliter. Sed potes hæc postea diligentius examinare."

There is no other remark on this subject in the following, except under the 21st March, when the following note was written by Tycho's artificer, Hans Crol (Johannes Aurifaber), who took (or at least entered) the observations that evening :—

"*N.B.*—Confer altitudinem meridianam cum his circa 90 gradum, atque hinc erue parallaxin ♀. Nam refractione caret in ista altitudine citima iuxta 90 gradum."

This shows that the subject had attracted some attention, but unfortunately the matter seems never to have been taken up again. During the very long series of observations of Mars from June 14, 1593, to February 14, 1594, there was no attempt made to determine the parallax. Under December 11, 1594, there is a note (not written by Tycho) to the effect that Venus, being now at the greatest elongation from the Sun, has the same parallax as the Sun. But Venus was then in −15° declination, and could not be followed for many hours, as in 1590. In 1595 one of Tycho's pupils calculated the distance of Venus from the Earth, assuming the distance of the Sun from the Earth to be 1112, and that of Venus from the Sun 822, the angle at the Sun being 10°; the result was 343 semidiameters of the Earth, so that the parallax of Venus at an altitude of 12° was 9′ 20″.

On no other occasion does Tycho appear to have neglected to

* In the *Progymn.*, p. 472, he assumes the distance of the Sun = 1152 semidiameters of the Earth (π = 2′ 59″), because 576 is a Pythagorean number.

follow up a discovery revealed by his observations. It was his custom to compare the observations of Sun, Moon, and planets with the tables and ephemerides, and the ledgers of observations are full of these comparisons, often accompanied by comments on the differences found. His journals and letters show how early he perceived the important results of his observations of the Moon, from the first moment that these observations showed anything different from the theories of the ancients. Similarly in the case of his discovery that the epicycles of the planets (especially that of Mars) varied in size, which he perceived had something to do with the solar excentricity, though it was reserved for Kepler to give a full explanation of the phenomenon. But had Tycho lived longer, he would probably have looked more closely into the subject of the parallax of the nearest planets.

Preliminary Comparison with Observations of the Tables of the Four Great Satellites of Jupiter.

By Professor R. A. Sampson, F.R.S.

The *Tables of the Four Great Satellites of Jupiter*, recently published by Durham Observatory, rely for modern observations upon photometric eclipses observed at Harvard College Observatory between 1878 and 1903. The residuals which these observations leave when compared with the old theory, pieced out with known and unknown corrections, are set forth fully in *Harvard Annals*, vol. lii., part 2. The elements upon which the theory of my new tables rest are determined by the values of the unknowns there given. Hence it was to be inferred that the residuals outstanding when the observations are compared with these tables would be generally the same as those shown for the same observations in *Harvard Annals*, but subject to some qualifications. It might, at first sight, appear that a more complete and consistent theory, as I believe it to be, than has hitherto been given, should reduce the errors somewhat; but there is an offset to this: the elements were determined by equations which assumed that the old theory would stand unchanged except in certain leading terms, so that if a revision of theory alters any secondary terms they fit in less well than before. The most striking instance of this occurs in the expression for the latitude of satellite IV, where the term which I give (Introduction, p. 16) as $-\cdot 01456 \sin h_3'''$ stands in Damoiseau with coefficient ·020478, and this difference alone, which I think could not possibly have been allowed for, is capable of altering the time of an eclipse, within the latitudes discussed below, by $\pm 140^s$. And again, the residuals in *Harvard Annals* rest for each satellite upon a separate determination of certain quantities, such as the semidiameter of Jupiter, the inclination and node of his equator,

at the values most suitable to the eclipses of that satellite, whereas, before a new theory could be constructed, a single compromise value for each quantity had to be adopted, and this necessarily meant some sacrifice of agreement. The burden of this compromise fell chiefly upon satellite III, and is quite clear in the comparison.

I propose, as opportunity allows, to recalculate all existing photometric eclipses from the new tables as a means of criticising and hereafter improving them, but meanwhile, as their early publication was desirable, I have contented myself with doing this for satellite IV alone, and, for the others, with verifying that no oversight had been made in putting the tables together, and that the residuals for good unselected eclipses do in fact come out as nearly the same as could be expected with those shown in *Harvard Annals*.

From the results, it is apparent that there still remains a good deal to do in the problem of Jupiter's satellites. I would quote from the conclusion of my memoir in *Harvard Annals* (lii. p. 343): —"Upon review of the whole, I am of opinion that the confusion of theory and tables was too great to allow a conclusive treatment of these observations at a single step. . . . The best course will be to construct from . . . clear data, a complete, consistent, gravitational theory, whence the masses may be derived and a set of tables calculated, with which at some future date the available observations may be compared. . . ."

It may perhaps be felt that, in view of the large divergence of theory and observation in some cases, the tables have been given a form more extended and more elaborate than necessary. I cannot admit this. A set of tables is built for the future. Strictly speaking, besides prediction, which would be equally useful even if quite rough, their only *raison d'être* is the criticism of the constants and formulæ upon which they are founded, and they can serve this purpose only up to that point of refinement to which they have themselves been calculated. But, besides this principle, that calculation should be refined well beyond the observations with which it is compared, I have given reasons for believing that the most notable of the divergences alluded to above, and indeed the greater number of them, medium as well as large, are due to some physical cause, probably in Jupiter's surface, which requires further examination. What we want is more observations to track this down, and it is to be hoped that amateurs may in future systematically make these photometric observations of the eclipses, which require no more time at the telescope than the traditional form, and are the only ones that are now of value. The following schedule contains the comparison of the tables for eclipses. The second column contains the observed time as found from the mean of my own method of reading the observations and that practised at Harvard. It should be clearly understood what this represents. The observation is actually a light-curve; from this curve, by rules which I have given at length in *Harvard Annals*, and in more condensed form in the tables, the instant of

half-brightness is determined, and this is taken to occur when the centres of the satellite and the Sun are in line with Jupiter's rim. The third column is the time as given by the tables with fractions of 1^s rounded off. The fourth is the amount by which the tabular time falls short of the observed. The fifth is the mean of the residuals derived from the Durham and Harvard readings, as shown in *Harvard Annals*, vol. lii. p. 310, and quoted here for comparison; the weight factor which depressed some of these residuals has been removed to make the comparison a fair one.

SATELLITE I.

Date.				Observation.			Calculation.			O − C.	(O − C)H.
				h	m	s	h	m	s	s	s
1878	VI	29	D	18	43	45	18	44	7	− 22	− 25
1879	XII	7	R	13	4	57	13	4	15	+ 42	+ 36
1880	X	24	R	15	54	38	15	54	38	0	− 1
1881	I	3	R	11	12	53	11	12	44	+ 9	+ 4
1882	X	30	D	20	34	50	20	34	56	− 6	− 9
1883	II	26	R	12	52	38	12	52	34	+ 4	− 3
1884	IV	1	R	12	35	41	12	35	36	+ 5	+ 4
1885	III	10	R	20	47	34	20	47	43	− 9	+ 6
1886	IV	23	R	16	34	27	16	34	37	− 10	+ 5
1887	IV	19	D	15	20	11	15	20	15	− 4	− 6
1888	IV	12	D	21	52	17	21	52	2	+ 15	+ 6
1889	VII	13	R	14	50	12	14	50	20	− 8	− 3
1890	VI	30	D	17	26	19	17	25	33	+ 46	+ 46
1891	IX	28	R	16	30	47	16	30	45	+ 2	+ 6
1892	IX	30	D	17	43	1	17	43	19	− 18	− 14
1893	X	5	D	15	42	50	15	42	49	+ 1	+ 4
1894	X	17	D	15	29	36	15	29	36	0	− 1
1895	IV	5	R	15	41	21	15	41	22	− 1	− 5
1896	V	25	R	13	40	23	13	40	31	− 8	− 6
1897	IV	28	R	14	18	42	14	18	59	− 17	− 6
1898	II	28	D	16	16	36	16	16	34	+ 2	+ 2
1899	IV	18	D	19	29	11	19	28	56	+ 15	+ 12
1900	IV	30	D	18	46	14	18	46	4	+ 10	− 5

SATELLITE II.

Date.				Observation.			Calculation.			O − C.	(O − C)III.
				h	m	s	h	m	s	s	s
1878	VII	7	D	20	6	31	20	6	48	− 17	+ 10
1879	VIII	20	D	13	58	4	13	58	25	− 21	+ 5
1880	X	23	R	18	36	11	18	36	27	− 16	+ 8
1881	XI	25	R	21	52	37	20	53	0	- 23	− 16
1882	XI	19	D	17	58	42	17	58	26	+ 16	− 9
1883	IV	14	R	13	48	9	13	48	9	0	0
1884	V	9	R	12	45	32	12	45	39	- 7	- 17
1885	III	14	R	17	23	55	17	23	58	− 3	− 7
1886	II	18	D	18	18	42	18	18	44	- 2	+ 22
1887	III	30	D	21	35	23	21	35	18	+ 5	- 1
1888	IV	17	D	17	14	34	17	14	29	+ 5	− 13
1889	VIII	10	R	14	45	50	14	45	19	+ 31	+ 6
1890	VII	17	D	16	47	17	16	47	18	- 1	- 6
1890	VIII	4	R	14	5	27	14	4	40	+ 47	- 1
1891	VIII	19	D	18	48	8	18	48	12	− 4	- 8
1892	I	5	R	12	31	36	12	31	27	+ 9	+ 12
1893	IX	7	D	18	49	38	18	48	50	+ 48	+ 39
1894	IV	5	R	13	41	28	13	41	24	+ 4	− 11
1895	III	5	R	16	14	45	16	14	52	− 7	+ 3
1896	XII	14	D	22	39	9	22	39	19	- 10	− 29
1897	I	26	D	14	16	10	14	16	23	- 13	− 12
1898	IV	26	R	14	25	38	14	26	20	− 42	- 37
1899	V	4	R	17	42	36	17	43	8	− 32	− 14
1900	III	16	D	22	28	31	22	28	22	+ 9	+ 28

SATELLITE III.

Date.				Observation.			Calculation.			O − C.	(O − C)III.
				h	m	s	h	m	s	s	s
1878	X	16	D	12	55	47	12	55	36	+ 11	− 6
1879	VII	15	D	17	35	17	17	35	22	− 5	- 34
			R	21	10	1	21	9	25	+ 36	− 6
1880	VIII	5	D	19	5	18	19	5	29	− 11	- 43
1881	X	9	D	20	52	55	20	54	0	− 65	− 16
			R	22	36	4	22	35	1	+ 63	+ 29
1882	IX	18	D	21	20	29	21	20	50	− 21	1
1883	I	18	D	17	8	39	17	8	21	+ 18	− 39
			R	19	49	59	19	50	1	− 2	- 19
1884	III	16	D	12	22	27	12	22	16	+ 11	+ 1
			R	15	57	4	15	56	59	+ 5	+ 11

SATELLITE III.—*continued.*

Date.			Observation.				Calculation.				O − C.	(O − C)H.
				h	m	s	h	m	s		s	s
1885	III	2	R	18	53	57	18	53	27		+ 30	+ 36
1886	V	6	D	13	57	0	13	56	50		+ 10	− 20
			R	16	44	22	16	44	34		− 12	− 8
1887	IV	22	R	18	42	31	18	42	55		− 24	− 17
1888	IV	7	D	19	32	9	19	31	0		+ 69	+ 52
			R	21	15	25	21	16	47		− 82	− 43
1889	VIII	29	D	13	39	59	13	39	55		+ 4	− 11
1890	IX	20	D	13	15	55	13	16	9		− 14	+ 3
1891	XII	30	D	10	34	31	10	34	29		+ 2	− 8
			R	13	41	34	13	41	30		+ 4	+ 17
1892	IX	20	D	15	34	15	15	33	46		+ 29	− 1
			R	17	54	51	17	52	56		+115	+128
1893	X	12	D	16	59	46	17	0	12		− 26	+ 3
			R	18	42	24	18	40	55		+ 89	+ 22
1894	IX	21	D	17	12	59	17	13	58		− 59	− 18
			R	19	47	2	19	45	44		+ 78	+ 30
1895	III	5	D	13	0	49	13	0	46		+ 3	+ 20
1895	III	5	R	16	1	11	16	1	0		+ 11	− 23
1896	II	19	R	19	45	28	19	45	28		0	− 6
1897	II	4	D	19	8	44	19	8	23		+ 21	− 7
1899	III	20	D	16	45	22	16	45	22		0	+ 22
			R	18	34	0	18	34	32		− 32	+ 16
1900	VII	6	R	16	55	40	16	56	4		− 24	− 7

SATELLITE IV.

Date.			Observation.				Calculation.				O − C.	(O − C)H.	Wt.
				h	m	s	h	m	s		s	s	
1878	IX	1	R	15	20	7	15	17	37		+150	+ 69	3
1878	XI	7	D	11	23	0	11	21	23		+ 97	+ 33	3, 3
1879	VIII	2	R	18	37	45	18	38	17		− 32	+ 74	4
1879	X	25	R	13	31	10	13	32	47		− 97	+ 36	3
1883	IX	18	D	21	41	48	21	41	22		+ 26	+ 20	5
1883	X	5	R	19	18	34	19	19	34		− 60	− 93	5
1883	XI	24	D	21	40	8	21	39	20		+ 48	+ 24	5
1883	XII	28	R	13	51	14	13	50	18		+ 56	+ 30	4
1884	III	4	R	14	17	55	14	16	48		+ 67	+ 42	5
1884	IV	23	D	15	56	56	15	56	53		+ 3	− 31	5
1884	V	10	R	14	43	32	14	42	54		+ 38	+ 9	5
1886	II	22	D	16	25	33	16	27	46		−133	− 21	5

SATELLITE IV.—*continued.*

Date.			Observation.				Calculation.			O−C.	(O−C)H.	Wt.
				h	m	s	h	m	s	s	s	
1886	IV	30	R	18	11	14	18	4	29	+405	−301	4
1889	VIII	18	D	14	26	45	14	29	30	−165	+ 16	5
1890	VII	19	D	14	57	30	14	58	16	− 46	+ 7	4
1890	VIII	5	R	13	42	18	13	40	37	+101	+ 37	5
1890	IX	24	D	15	28	32	15	29	20	− 48	− 12	5
1891	VII	6	R	17	5	11	17	6	29	− 78	− 8	5
1891	IX	28	R	11	51	49	11	53	51	−122	+ 4	5
1895	II	19	D	13	27	25	13	33	13	−348	−116	4
1895	X	28	D	19	43	30	19	43	0	+ 30	+ 28	5
1896	I	3	D	19	41	46	19	41	16	+ 30	+ 7	5
1896	III	27	D	13	51	44	13	51	44	0	− 26	5
1896	IV	13	R	12	41	13	12	41	18	− 5	− 26	4
1896	VI	2	D	14	4	45	14	4	13	+ 32	+ 10	5
1896	VI	19	R	12	58	42	12	58	58	− 16	− 45	5
1896	XII	3	D	20	8	59	20	8	1	+ 58	+ 63	5
1896	XII	20	R	18	50	55	18	49	43	+ 72	+ 6	5
1897	II	25	R	18	36	21	18	35	16	+ 65	− 22	4
1897	XI	8	D	20	34	16	20	35	43	− 87	+ 13	4
1898	I	9	D	20	36	8	20	39	17	−189	+107	4
			R	22	51	58	22	49	3	+175	− 31	4
1901	IX	28	D	12	33	16	12	35	17	−121	− 71	5
1902	VI	6	D	19	17	1	19	17	18	− 17	+ 24	3
1902	VI	23	R	18	1	3	18	0	5	+ 58	+ 14	5
1902	IX	15	R	12	48	10	12	48	26	− 16	− 42	5
1903	VII	30	D	17	54	30	17	53	51	+ 39	− 78	3
1903	X	22	D	13	19	26	13	18	56	+ 30	−135	5
1903	X	22	R	16	41	47	16	43	14	− 87	+ 57	5

Besides the eclipses, I have made calculations of the other phenomena: of these I shall give only a specimen.

Mr. Innes gives in *Mon. Not.*, lxix. p. 517 (1909) the observations shown in the third column below; the fourth column shows the corresponding times by my tables.

Date.	Phenomena.	Observation.		Calculation.	O−C.
		h m s	s	h m s	s
1908 Feb. 2	I. Oc. In.	8 41 56·3	±1·1	8 42 34	− 38
3	I. Tr. In.	6 0 39·6	±2·7	6 2 30	−110
3	I. Sh. In.	6 10 13·1	±4·1	6 9 47	+ 26
3	I. Tr. Eg.	8 17 35·3	±3·3	8 20 11	−156
3	I. Sh. Eg.	8 25 16·5	±5·6	8 27 24	−127

To determine what is the nature of these divergences we have from the observations—

		h	m	s			h	m	s
Transit:	In.	6	0	40	Shadow:	In.	6	10	13
	Eg.	8	17	35		Eg.	8	25	17

whence, taking half the sum and half the difference of each pair,

	h	m	s	h	m	s
Conjunction + Red^{n}. to Middle	7	9	8	7	17	45
Semiduration . . .	1	8	28	1	7	32

Now it appears that the difference of the Reductions to Middle by my tables is only 5^{s}; hence the difference of the times of conjunction, which measures the annual parallax, is $-8^{m}\,42^{s}$; or in angle, in proportion with the synodic motion, $-1^{\circ}\cdot2295$. But from data in the *Nautical Almanac* the annual parallax was actually $-1^{\circ}\cdot0398$. So that somewhere among these four observations is a discrepancy of $0^{\circ}\cdot19$, or in time $1^{m}\,22^{s}$, which no theory could remove. Similarly, I find that the differences between the semiduration—which are near their maxima—amount to only 2^{s} in place of 56^{s}.

I drew similar conclusions from other comparisons, and generally it appeared that the phenomena other than eclipse are not capable at present of affording any convincing test of the accuracy of the theory.

As the construction of a set of tables is not undertaken very often, a few particulars about it may be of interest. The theory was first completed, and then two schedules were drawn up, one containing the value at epoch and the mean motion of every argument, and the other, a list of all the inequalities it was proposed to tabulate. Then a memoir was made and added to from time to time, known as the "Construction," which was intended as a bridge from these schedules to the precepts given to the two computers, Mr. Carpenter and Mr. Doak. The Construction quoted all the numbers that were used in each precept, and contained any discussion that was necessary for myself in working out the step dealt with. Thus, as an example of the calculation of the arguments, I find in the Construction the following entry:—

"1900·0 $\quad d_{12}=43^{\circ}\cdot04906 \quad n_1-n_2=102^{\circ}\cdot114\ 230\ 763.$

(Precept 16, 1908 May 21) Find first the period ($3^{d}\cdot54$ approx.), then the number of days the value of d_{12} at epoch is worth. Next construct the residues for each number of whole days from 1^{d} to 366^{d}.

Then find the number of complete periods and residue in 365^{d} and 366^{d}, and construct epoch at Jan. 0·0 for each year from 1850 to 2000."

This was expanded as follows for Mr. Carpenter in a sheet that was affixed to his calculations :—

"Tables. Precept 16. 1908 May 21.
Epochs for argument d_{12}.
The motion of the argument in $1^d = 102^\circ{\cdot}114\ 230\ 763$.
Find the period (approx. $3^d{\cdot}54$) by dividing this into 360°.
The value of the argument at epoch 1900·0 is $43^\circ{\cdot}04906$; find the equivalent of this in days by dividing by the motion. These two should be verified carefully by independent methods; the period should be found to as many decimals as contracted long division allows. The value of long. at epoch 1900·0 should be found to $0^d{\cdot}000\ 001$.

Find the value of the argument after the lapse of 1, 2 . . . . 366 whole days as follows (suppose the period is $3^d{\cdot}543$ as an illustration).

Days.	Argument.	Multiples.
1	1·000	
2	2 000	
3	3·000	
		3·543
4	0·457	
5	1·457	
6	2·457	
7	3·457	
		7 086
8	0 914	
9	1·914	

Each time the sum would outrun the period you subtract the period; thus, instead of 4 you put 0·457, in place of 4·457 you put 0·914, and so on. The best way will be to form multiples of the period until they exceed 366^d, and use these. These multiples to be formed with the help of an addition slip or Brunsviga. The argument is wanted to 0·00001, and there are about 100 multiples, hence you must form the multiples with 0·0000 0001 in order to have a guard figure.

Next, using two more figures, find the multiples for 365^d and 366^d, and their residues correct to 0·0000 0001, and using addition slips build up the value of the argument at Jan. 0·0 of each year from 1900 forwards to 2000, and backwards to 1850, correct to 0·00001, after cutting off the guard figure."

This was done by Mr. Carpenter without further direction in 11 hours.

As an example of the calculation of an inequality we have in the Construction, relating to the equations of latitude, concluding a discussion of the accuracy necessary :—

"Hence we have the following precepts for the main equations for ζ:

(Pre. 69 / 1908 Dec. 15)

Sat. I.: $\delta = 1^\circ{\cdot}017445$.
h_0 factor for diff. ·0060228.
factor for eqn. ·339180.
400 entries,

take $(n+\frac{1}{2})h = 0^\circ{\cdot}01$, then to $0'{\cdot}1$. Take cos $(n+\frac{1}{2})h$ to ·0001, Δ to ·000 001.

17 entries to column, 2 columns to page.
recompute head of each column."

This was given to Mr. Doak in the following form:—

"Tables. Precept 69. 1908 Dec. 15.

Sat. I. Table of equation $+ \cdot 339180 \sin h_0$.

Take 400 intervals each $0^d{\cdot}005$, in each of which the motion of argument h_0 is $\delta = 1^\circ{\cdot}017445$.

Set up $\frac{1}{2}\delta$ upon the product-table of a Brunsviga machine and δ upon the levers, and by giving 400 turns form $(n+\frac{1}{2})\delta$ from $n = 0$ to $n = 400$.

Copy these into column 4 to $0^\circ{\cdot}01$, but at the head of every column copy also into the margin to $0^\circ{\cdot}0001$.

Convert to ° ′ to $0'{\cdot}1$ for column 4.

In column 4 take out natural cosines to ·0001.

Set up on Brunsviga multiplier ·0060228 and multiply into the nat. cosines. Copy the products in column 3 (Δ) to ·00001, but in doing so the initial zeroes may be dropped. These are the differences of the table.

Construct the table to ·00001 by adding the differences. But start each column, not from the sum of the differences, but by computing separately $\cdot 339180 \sin n\delta$. From $(n+\frac{1}{2})\delta$ as written in the margin subtract $\frac{1}{2}\delta = 0^\circ{\cdot}5087$; convert to ′ ″ up to $0''{\cdot}1$. Take out L sin to 6 figures. Add log $\cdot 339180 = 9{\cdot}530417$ and take out antilog. to ·00001. Enter this in column 2 as the equation at the head of each column.

Write in pencil or otherwise in the margin the corresponding sum of differences.

Model on next page."

This was done by Mr. Doak in 14 hours, my estimate of the time necessary having been 18 hours.

There were in all 200 of these precepts, several of them subdivided. They ranged in length from $\frac{1}{2}$ hour to 70 hours. As a rule I was able to estimate the time each would take within one-tenth to one-fifth of the whole.

The calculation of these precepts was the heaviest part of the work, but the crucial point was their verification, the addition of the necessary tabulation constants, and the formation of a MS. for the printer. The whole of this I did at first myself, but after completing Sat. I and Sat. II I became so familiar with the

precautions necessary that I was able to pass over safely some of the more straightforward tables of Satellite III and Satellite IV to Mr. Carpenter. First, each calculation was examined to see that the precept had been followed and the correct numbers carried into it; then it was read through to see that there were no rough places in its differences. Then I wrote it out as a table, adding to each entry the determined tabulation constant. Then from and upon my manuscript I formed the difference column. Then I verified the whole by reconstructing each entry from its predecessor and the differences, keeping the entry concealed from view until I had read to myself its reconstructed value.

Three first proofs were pulled for each sheet by Messrs. Neill & Co.; these proofs were very accurate, a sheet of eight pages often yielding only two or three corrections: the first I read against the MS.; the second I corrected and returned to Messrs. Neill for a revise; the third Mr. Carpenter read independently against the MS., and his corrections were a check upon my own, and if he found any new ones they were inserted in the revise. It is hoped that with these precautions the tables will be found to be almost free from numerical errors.

On the Formulæ for the Comparison of Observed Phenomena of Jupiter's Satellites with Theory.

By W. de Sitter, Assoc. R.A.S.

1. The comparison of an observed eclipse, occultation, transit or shadow-transit of a satellite of Jupiter with the theory can be effected on two fundamentally different methods. The time of the phenomenon, *i.e.* the time at which the coordinates of the satellite satisfy certain conditions, can be predicted from the theory, and this predicted time can then be directly compared with the observed time. But the *prediction* is not necessary for the comparison. It is sufficient to compute by the theory the coordinates *at the observed time*, and then verify whether the condition is satisfied. The formulas for the application of this second method will be developed in the following paper.

Unless special tables for the prediction of phenomena have been constructed, based on the theory which is to be tested by the observations, the computation of the coordinates at a given time is much easier than the computation of the time for given coordinates. My theory of the satellites (*Proceedings Amsterdam Academy*, March 1908) is, in my opinion, still of too preliminary a character to warrant the construction of tables for the prediction of phenomena based upon it. But for the second method of comparison all that is wanted are tables facilitating the computation of the coordinates at a given time. These are much more

easily constructed, and, moreover, can also be used for observations of the satellites away from conjunction (photographs or heliometer-observations). I have therefore computed such tables, and I am at present with their aid comparing Mr Innes's observations (*M.N.*, 1909 April and November) with my theory.

But even when tables for the prediction of phenomena exist, the method of coordinates offers some advantages over the method of times. Firstly, the equations of condition are simpler and more direct, especially so for the latitudes. Then it may happen that a phenomenon cannot be predicted at all, for the reason that, according to the theory, it ought not to occur. If it occurs nevertheless, and is observed, the corrections to the theory can only be derived by the method of coordinates. A case of this kind will be treated at the end of this paper.

2. Let a system of rectangular axes of coordinates be placed with its origin O in the centre of Jupiter, the axis of z being the axis of rotation of the planet, positive to the north. The axis of x is the projection of the line Sun-Jupiter, or Earth-Jupiter on the plane of Jupiter's equator. It is positive towards the position of the *Satellite*. The positive axis of y is reached from the positive axis of x by a rotation of 90° in the direction of motion of the satellite. The y coordinate of the satellite (S) is thus always small, and negative at ingress, positive at egress, for all phenomena. Its x is the projection of its radius vector on the axis of x, and will be called r'.

The equatorial and polar semidiameters of the planet are b and $b/(1+\rho)$. The equation to its surface is thus

$$x^2 + y^2 + z^2(1+\rho)^2 = b^2.$$

The coordinates of a point A (Sun or Earth) are

$$X = \mp \Delta \cos B \qquad Y = 0 \qquad Z = \Delta \sin B \qquad \begin{pmatrix} -\text{Ecl., Occ.} \\ +\text{Trans., Shad.} \end{pmatrix}$$

We have now to find the condition that the line joining A and S is tangent to the surface of Jupiter.

In the case of transits and shadow-transits we must take account of phase. We have then to consider two bodies, of which the coordinates are

$$\text{A}: \quad X = \Delta \cos B \qquad Y = 0 \qquad Z = \Delta \sin B$$
$$\text{A}': \quad X' = \Delta' \cos B' \cos(L' - L) \quad Y' = \Delta' \cos B' \sin(L' - L) \quad Z' = \Delta' \sin B'.$$

The problem can be split up in two, viz.—

(α) To find the "terminator," *i.e.* the curve of contact of Jupiter's surface with the tangent cone whose apex is A'.

(β) To find the projection of this terminator on the plane $x = r'$

from the point A. The coordinates y, z of the satellite must then satisfy the equation to this projection.

For eclipses and occultations A and A′ are the same, viz. the Sun for eclipses and the Earth for occultations. For transits and shadow-transits they are also the same (unless in exceptional cases, for which see art. 11), when y and Y′ are of the same sign. When they are not of the same sign we must take account of phase. We have then—

	$L'-L>0$	$L'-L<0$	
Trans. in.	A′ = Sun	A′ = Earth	A = Earth throughout.
eg.	Earth	Sun	
Shad. in.	Earth	Sun	A = Sun throughout.
eg.	Sun	Earth	

3. The problem can be simplified by the following consideration. Suppose the problem to be solved, and let *all* z-coordinates be increased or diminished in the same ratio. The surface of the planet will become an ellipsoid with a different ellipticity, and the points A and S will be displaced, but the line AS joining their new positions will still be tangent to the modified surface. Consequently we can begin by solving the problem for a spherical planet ($\rho = 0$), provided we adopt for the z coordinates of A and S values Z_1 and z_1, such that they are reduced to the true values by the subsequent division by $(1+\rho)$. Thus

$$Z_1 = Z(1+\rho) \qquad z_1 = z(1+\rho).$$

We will continue to write z for z_1. In the final formulas we must then replace z by $z(1+\rho)$. The coordinates X and Y, x and y are not changed. Consequently

$$\Delta_1 \cos B_1 = \Delta \cos B \qquad \Delta_1 \sin B_1 = (1+\rho)\Delta \sin B$$

from which

$$\tan B_1 = (1+\rho) \tan B$$

$$\cos B_1 = \frac{\cos B}{\sqrt{1+\rho(2+\rho)\sin^2 B}} \qquad \sin B_1 = \frac{(1+\rho)\sin B}{\sqrt{1+\rho(2+\rho)\sin^2 B}} \qquad (1)$$

and similarly for B'_1.

4. In the simplified, or spherical problem, the terminator (problem (α) above) is a circle in a plane perpendicular to the line OA′. Its centre is on this line, at the distance b^2/Δ_1 from the origin. Its radius is $b\sqrt{1 - b^2/\Delta_1^2}$.

Now the ratio b/Δ_1 is of the order of 10^{-4}. Consequently b^2/Δ_1 can be neglected in comparison to Δ_1 and b^2/Δ_1^2 in comparison to unity. We can thus with sufficient accuracy say that the terminator is a circle with radius b, lying in a plane through the origin and perpendicular to OA′.

5. This circle must now be projected from the point A on the plane $x = r'$. This we do in two steps. First we find the projection on the plane $x = 0$. This is an ellipse, which will be called E_0. The projection on the plane $x = r'$ is then also an ellipse, whose centre is the projection of the centre of E_0, and whose axes are parallel to those of E_0. The ratio of the axes also is the same, both axes being derived from those of E_0 by multiplying them with

$$1 \pm \frac{r'}{\Delta_1 \cos B_1} \qquad \begin{pmatrix} + \text{Ecl., Occ.} \\ - \text{Trans., Shad.} \end{pmatrix}.$$

6. First take the simplest case $A = A'$. One of the axes of E_0 is then along the axis of z, and is the projection of the diameter of the terminator which lies in the plane $y = 0$. The extremities of this axis are thus the points of intersection of the line $x = 0$ with the lines joining the point

$$X = \Delta_1 \cos B_1 \qquad Z = \Delta_1 \sin B_1$$

to the two points

$$x_1 = \mp b \sin B_1 \qquad z_1 = \pm b \cos B_1.$$

We find

$$z = -\frac{b^2}{\Delta_1} \sin B_1 \pm b \sec B_1.$$

The length of the z-semi-axis of the ellipse E_0 is thus $b \sec B_1$. The coordinates of the centre are

$$y_0 = 0 \qquad z_0 = -\frac{b^2}{\Delta_1} \sin B_1.$$

Now $\sin B_1$ never exceeds 1/15. The maximum value of z_0 is therefore 1/150000, and can be neglected. This is equivalent to saying that, instead of projecting the terminator from the point A as centre, we may project it on $x = 0$ by lines parallel to OA. The other axis is then along the axis of y, and its length is b. The equation to E_0 is thus

$$y^2 + z^2 \cos^2 B_1 = b^2 \qquad (2)$$

7. Now take the case that A differs from A'. The terminator is a circle in the plane (a') perpendicular to OA'. This must be projected on the plane $x = 0$. We first project it on the plane (a) perpendicular to OA, which projection is an ellipse called E'_0. This is then projected on $x = 0$ to give E_0. The latter operation is similar to the one applied to the terminator in the preceding article, and (if we adopt again parallel instead of central projection) consists of leaving the y-coordinates unaltered and multiplying the z's by $\sec B_1$.

In finding E_0' we can also use parallel projection. The error which is thereby introduced is $b^2 \sin \delta/\Delta_1$, where δ is the angle between the planes (α) and (α'). The maximum value of $\sin \delta$ is about 1/5. The maximum error is therefore 1/50000, which can still be neglected, as it amounts to less than 0″·0005 of geocentric arc. The ellipse E'_0 has its longer axis along the line of intersection of the planes (α) and (α'), and equal to b. Its shorter axis is $b \cos \delta$. Its equation thus is

$$\eta^2 + \zeta^2 \sec^2 \delta = b^2,$$

if η and ζ are coordinates in the plane (α), the axis of η being the intersection of the two planes. The angle between this and the axis of y is v, where v and δ are determined by the equations

$$\left.\begin{aligned} \cos \delta &= \sin B_1 \sin B'_1 + \cos B_1 \cos B'_1 \cos (L' - L) \\ \sin \delta \cos v &= \cos B_1 \sin B'_1 - \sin B_1 \cos B'_1 \cos (L' - L) \\ \sin \delta \sin v &= \cos B'_1 \sin (L' - L). \end{aligned}\right\} \quad (3)$$

The equation to E'_0 on the axes of y and z' [z' being the projection of z on the plane (α)] is thus

$$(y \cos v - z' \sin v)^2 + (y \sin v + z' \cos v)^2 \sec^2 \delta = b^2.$$

To project on $x = 0$ we must put $z' = z \cos B_1$. The equation to E_0 thus becomes

$$y^2 + z^2 \cos^2 B_1 + \tan^2 \delta (y \sin v + z \cos B_1 \cos v)^2 = b^2. \quad (4)$$

8. We now introduce the compression ρ by writing $z(1 + \rho)$ for z, and substituting for $\cos B_1$ its value given by formula (1). The equations (2) and (4) thus become

$$y^2 + z^2(1 + c)^2 = b^2 \quad (5)$$

$$y^2 + z^2(1 + c)^2 + [Sy + Gz(1 + c)]^2 = b^2 \quad (6)$$

where

$$1 + c = \frac{(1 + \rho) \cos B}{\sqrt{1 + \rho(2 + \rho) \sin^2 B}}$$

$$S = \tan \delta \sin v \qquad G = \tan \delta \cos v.$$

9. The problem considered so far has also been solved by Bessel ("Über die Scheinbare Figur eines unvollständig erleuchteten Planeten," *Astron. Unters.*, i. p. 239). Bessel's treatment is entirely analytical. He derives his results for the compressed planet at once, without making use of the solution for the sphere. The principal advantage of the method used here, not counting its greater simplicity, seems to me to be that it is very easy to evaluate, and, if necessary, to take account of the neglected terms, which in

Bessel's method would entail considerable complications.* Bessel's formulas corresponding to our (5) and (6) are those numbered (12) and (15). In comparing, it must be remembered that Bessel's β and β' are geocentric or heliocentric latitudes, while our B and B′ are jovicentric, thus $\beta = -B$, $\beta' = -B'$, and consequently

$$\begin{aligned} &\text{Bessel's} && d = \text{our} && \delta \\ &\quad ,, && \sin w = ,, && \sin v \\ &\quad ,, && \cos w = ,, && -\cos v. \end{aligned}$$

Further, Bessel's y is positive in the direction of increasing geocentric longitudes, *i.e.* opposite to our y. Bessel finds the projection on the sphere having A at its centre, or, as he takes the arc equal to the sine and the tangent, on our plane (α). Thus

$$\begin{aligned} &\text{Bessel's } u' = \text{our} - y \\ &\quad ,, \qquad v' = ,, \; z \cos B. \end{aligned}$$

Further, Bessel uses excentricity instead of compression. Thus

$$\text{Bessel's } 1 - e^2 = \text{our } 1/(1+\rho)^2.$$

Taking due account of these identifications, we can easily verify that Bessel's equations (12) and (15) are identical with our (5) and (6).

10. It now remains to project our ellipse E_0 on the plane $x = r'$. All we have to do is to increase its dimensions in the ratio $1 \pm r'/\Delta \cos B$, and to find the projection of the centre. The coordinates of the latter are

$$y_0 = 0 \qquad z_0 = \mp r' \tan B.$$

Thus, if we take a new origin of coordinates in the centre, we have

$$z' = z - z_0$$

and we find for our final formulas

$$y^2 + z'^2(1+c)^2 = C^2 \tag{7}$$

$$y^2 + z'^2(1+c)^2 + [Sy + Gz'(1+c)]^2 = C^2 \tag{8}$$

$$z' = z \pm r' \tan B \tag{9}$$

$$C = b\left(1 \pm \frac{r'}{\Delta} \sec B\right) \left\{ \begin{matrix} +\text{Ecl., Occ.} \\ -\text{Trans., Shad.} \end{matrix} \right. \tag{10}$$

$$1 + c = \frac{(1+\rho)\cos B}{\sqrt{1 + \rho(2+\rho)\sin^2 B}} \tag{11}$$

The formula (8) must be used when A and A′ are different,

* In fact, the principal object of the present investigation was to decide whether the accuracy of Bessel's result was sufficient for my purpose.

i.e. for transits and shadow-transits when phase must be taken into account; the formula (7) in all other cases (see art. 2).

11. From the formulas (3) we find easily

$$\tan\delta\sin v = \frac{\tan\Psi\sec B_1}{1+\tan B_1\tan B'_1\sec\Psi},$$

$$\tan\delta\cos v = \frac{\sin(B'_1-B_1)\cos\Psi+2\sin B'_1\cos B_1\sin^2\frac{1}{2}\Psi}{\cos(B'_1-B_1)\cos\Psi+2\sin B'_1\sin B_1\sin^2\frac{1}{2}\Psi},$$

where Ψ has been written for $(L'-L)$.

We must now substitute B and B′ for B_1 and B'_1 by means of the formulas (1). The largest values of the sines and tangents are—

B and B′	1/15
B′ − B	1/70
Ψ	1/5.

Further $\rho = 1/15$.

Neglecting quantities of the order of 10^{-5}, we find, after some simple reductions, the following formulas, depending only on Ψ, B′ − B and B′:—

$$S = \tan\Psi - (1+\rho)^2\tan^2 B'\tan\Psi(\sec\Psi-\tfrac{1}{2}) - (1+\rho)^2\tan B'\tan(B'-B)\tan\Psi(\sec\Psi-1) \quad (12)$$

$$G = (1+\rho)\tan(B'-B) + (1+\rho)\sin B'\cos B'(\sec\Psi-1)$$

$$\Psi = L'-L$$

For transits: L′ = Sun, L = Earth,

shadow-transits: L′ = Earth, L = Sun.

The terms after the first are very small in both equations. They can be presented under several slightly different forms; the form chosen here seemed to be the most practicable for tabulating. In most cases they will be entirely negligible. Very often it will be allowable to neglect the whole of G, thus taking account of phase only in the coordinate y.

To decide whether for a given phenomenon phase should be computed or not, the following rule can be given. Plot on the plane of y, z' the point P whose coordinates are S, G (or, if these have not yet been computed, we can take $\tan\Psi$ and $\tan(B'-B)$, or even Ψ and $B'-B$). The line drawn through the origin, perpendicular to OP separates the part of the plane where phase should be computed from the part where it should not. The part where P is in has *no* phase. If we take $B'-B=0$ we find in this way the rule already given in art. 2.

12. The theory gives the radius vector r, longitude v, and latitude β of the satellite. The corresponding quantities Δ, $L+180^\circ$ and B for the body A must be computed from the data given in the national ephemerides.

The coordinates of the satellite are thus

$$\begin{aligned} x = r' &= r\cos\beta\cos\alpha \\ y &= r\cos\beta\sin\alpha \\ z &= r\sin\beta \end{aligned} \qquad (13)$$

Here α is the longitude counted from the axis of x, thus

$$\begin{aligned} \alpha &= v - \mathrm{L} && \text{for eclipses and occultations} \\ \alpha &= v - \mathrm{L} + 180^\circ && \text{for transits and shadow-transits} \end{aligned} \qquad (14)$$

For eclipses and shadow-transits L, B, and Δ refer to the Sun, for occultations and transits to the Earth.

13. The formulas (14) require a correction on account of the finite velocity of light. The value of α as given by these formulas is that satisfying the *geometrical* condition that the line joining A and S is tangent to the surface of Jupiter. But taking, for the sake of argument, the case of an eclipse immersion, the last ray of light which reaches the satellite is that which grazes the rim of the planet on its way. By the time this light reaches S, the whole system has moved on, and the geometrical line joining the Sun and the satellite now passes clear of Jupiter. In other words, the centre of coordinates, with reference to which the geometrical condition is satisfied, is not the true centre of Jupiter at the instant of immersion, but a point lying a distance ξ further back on the orbit. The longitude of the satellite referred to this point (which is the value to be used in our equation of condition) is larger than the true longitude, the excess being the angular value of ξ viewed from the satellite.

Let k be the constant of aberration. The time taken by the light to travel from Jupiter to the satellite is then $\tau = kr\cos(v - \mathrm{L})$. Further, $\xi = \tau\Delta\dfrac{d\mathrm{L}}{dt}$, and the correction to be applied to v in (14) is ξ/r or

$$\epsilon = k\Delta\frac{d\mathrm{L}}{dt},$$

the cosine being equal to unity.

A similar reasoning applies to the immersion, and to shadow-transits. In the case of occultations and transits, the ray of light to be considered is the returning ray from the satellite to the Earth. The sign of the correction is therefore changed. Collecting results we have, instead of (14),

$$\begin{aligned} &\text{Ecl.:} && \alpha = v - \Lambda + \epsilon \\ &\text{Occ.:} && \alpha = v - \mathrm{L} - \epsilon' \\ &\text{Shad.:} && \alpha = v - \Lambda + 180^\circ + \epsilon \\ &\text{Trans.:} && \alpha = v - \mathrm{L} + 180^\circ - \epsilon' \end{aligned} \qquad (15)$$

where

$$\epsilon = k\mathrm{R}_{\mathcal{U}}\frac{dl_{\mathcal{U}}}{dt}$$

$$\epsilon' = k\Delta\frac{d\lambda_{\mathcal{U}}}{dt} = k\Delta\frac{dl_{\mathcal{U}}}{dt} + k\mathrm{R}\sec\psi\cos(\odot - l_{\mathcal{U}})\left[\frac{d\odot}{dt} - \frac{dl_{\mathcal{U}}}{dt}\right].$$

To four places of decimals we have—

$$\epsilon = 0^\circ\cdot0025$$
$$\epsilon' = 0^\circ\cdot0052\cos(\odot - l_{\mathcal{U}}) + 0^\circ\cdot00047\,\Delta.$$

The notations used are those explained in the next article.

It may be remarked that a similar reasoning also holds in the case of observations of the satellite at other points of its orbit. Here we require the correction to be applied to the geometrically computed coordinate to give the observed value, *i.e.* the sign of the correction is reversed. The observed co-ordinate is thus—

$$y = \frac{r}{\Delta}\sin(v - \mathrm{L}) + kr\cos(v - \mathrm{L})\frac{d\mathrm{L}}{dt} = \frac{r}{\Delta}\sin(v - \mathrm{L} + \epsilon').$$

14. The fundamental plane of my theory is the mean equator of Jupiter, defined by its inclination and node on the mean ecliptic

$$\mathscr{I} = 2^\circ\ 12'\ 8''\cdot7 + 0''\cdot4231\,\mathrm{T}$$
$$\mathscr{N} = 336\ 24\ 24 + 48\cdot916\,\mathrm{T},$$

where T is the time counted in tropical years from 1900·0.

The jovicentric distances, longitudes and latitudes referred to this plane are denoted by the following lettters :—

for the satellite		r	v	β
,,	Sun	$\mathrm{R}_{\mathcal{U}}$	$\Lambda + 180^\circ$	B
,,	Earth	Δ	$\mathrm{L} + 180^\circ$	B

The longitudes and latitudes of Jupiter are—

heliocentric	$l_{\mathcal{U}}$	$b_{\mathcal{U}}$
geocentric	$\lambda_{\mathcal{U}}$	$\beta_{\mathcal{U}}$

Further, $\odot$ is the Sun's longitude and R the Earth's radius vector.

We have

$$\sin\beta_{\mathcal{U}} = \frac{\mathrm{R}_{\mathcal{U}}}{\Delta}\sin b_{\mathcal{U}}$$
$$\lambda_{\mathcal{U}} = l_{\mathcal{U}} + \psi \qquad \sin\psi = \frac{\mathrm{R}}{\Delta}\sin(\odot - l_{\mathcal{U}})\sec\beta_{\mathcal{U}} \tag{16}$$

(This ψ is approximately equal to Ψ (art. 11) for transits, and to $-\Psi$ for shadow-transits).

We might compute $\lambda_{\mathcal{U}}$ and $\beta_{\mathcal{U}}$ from the geocentric right ascension and declination given in the *Nautical Almanac*. The computation by (16) is, however, simpler, and, moreover, we require true values referred to the mean equinox, while the *N.A.* gives apparent R.A. and declination.

Further,

$$\begin{aligned} \sin \mathrm{B} &= \sin \mathscr{I} \sin \left(l_{\mathcal{U}} - \mathscr{N}\right) \cos b_{\mathcal{U}} - \sin b_{\mathcal{U}} \cos \mathscr{I} \\ \Lambda &= l_{\mathcal{U}} + \tfrac{1}{2}\mathscr{I}(b_{\mathcal{U}} - \mathrm{B}) \cos \left(l_{\mathcal{U}} - \mathscr{N}\right) \end{aligned} \qquad (17)$$

and similar formulas for B and L. Instead of the first, which is rigorous, it is generally sufficient to take the approximate formula,

$$\mathrm{B} = \mathscr{I} \sin \left(l_{\mathcal{U}} - \mathscr{N}\right) - b_{\mathcal{U}}.$$

The values of $l_{\mathcal{U}}$, $b_{\mathcal{U}}$, $\mathrm{R}_{\mathcal{U}}$ should be interpolated from the *N.A.* for the time t at which the observed ray left *the satellite*. The values of $\odot$ and R must be taken for the time t_0 at which it reaches the Earth. All longitudes must be true ones, and referred to the mean equinox. Thus:

$\odot$ = Sun's longitude from *N.A.* + aberration − nutation.

In the formula for C (10), and in those for $\beta_{\mathcal{U}}$ and ψ (16), Δ is the distance from the Earth at time t_0 to *Jupiter* at time t. The value of $\log \Delta$ interpolated from the *N.A.* for the time t_0 therefore requires the correction

$$\delta \log \Delta = -0{\cdot}434\ k\mathrm{R}_{\mathcal{U}} \sin \psi \frac{dl_{\mathcal{U}}}{dt}.$$

This correction is very small, and can generally be neglected. It is, however, easily tabulated, and can be taken into account without any trouble.

For the computation of the aberration time $t_0 - t$ this correction is always negligible. But here Δ is the distance from the Earth to the *satellite*, and therefore requires the further correction

$$\delta\Delta = r \cos (v - \mathrm{L}) \cos \mathrm{B}.$$

The corresponding correction to the aberration time is practically constant, and its value for the four satellites is

$$\pm 1^{\mathrm{s}}{\cdot}4 \quad \pm 2^{\mathrm{s}}{\cdot}2 \quad \pm 3^{\mathrm{s}}{\cdot}6 \quad \pm 6^{\mathrm{s}}{\cdot}3 \quad \begin{pmatrix} + \text{Ecl., Occ.} \\ - \text{Trans., Shad.} \end{pmatrix}$$

15. In all the preceding discussions the satellite, the Sun and the Earth have been treated as points. So far as the satellite

and the Sun are concerned the consideration of the effect of their finite dimensions is better deferred till the discussion of the observations. For the case of eclipses a very full discussion is given by Professor Sampson in *Harvard Annals*, vol. lii., pt. ii. pp. 166 *sqq.* But Mr Innes's method of observing is so different from that used at Harvard College, owing to the use of a high magnifying power, that it may be questionable whether the same considerations apply to both cases. The instant to which our formulas apply is that called by Professor Sampson the phase $k=0$.

For the Earth all we have to do is to correct L, B and Δ so that they refer to the observer instead of the centre of the Earth. The corrections on this account are with sufficient accuracy

$$\delta B = \pi \sin \beta_0$$
$$\delta L = -\pi \cos \beta_0 \sin (\lambda_0^{\bullet} - \lambda_\odot)$$
$$\delta \log \Delta = -0{\cdot}434\, \pi \sin 1'' \cos \beta_0 \cos (\lambda_0 - \lambda_\odot)$$

where π is the horizontal parallax of Jupiter, and λ_0 and β_0 are the ecliptic coordinates of the observer, which are computed by the usual formulas from the right-ascension = sidereal time, and declination = latitude. These corrections only affect the fourth decimal of a degree in B and L, and the sixth decimal of log Δ, and they can nearly always be neglected. For a given place of observation they can, however, easily be tabulated.

16. The theory, which it is proposed to compare with the observations, has been published in the *Proceedings of the Academy of Sciences at Amsterdam* for March 1908. The reader is requested to correct the following misprints in that paper:—

page 721, log a_4 should read 8·0998365
„ 725, the motion of τ is 51°·05711156
„ „ V „ 0·16618
„ „ V′ „ 0·16622
line 2 from bottom read $\delta p_i = -\kappa \sin \alpha$
„ 727, the signs of a_{21} and a'_{21} should be changed.

The coordinates v, β and r are computed by means of manuscript tables based on the data given in that theory. The latitudes β_i are then referred to the *mean* equator of Jupiter. But the formulas developed above suppose the *true* equator. A correction must thus seemingly be applied, which is

$$\delta\beta_i = -\Sigma_j \sigma_{ij} \gamma_j \sin (v_i - \theta_j).$$

Similar corrections should be added to B, and B computed by the formulas (17), viz.—

$$\delta \mathsf{B} = +\Sigma_j \sigma_{ij} \gamma_j \sin (\Lambda - \theta_j)$$
$$\delta B = +\Sigma_j \sigma_{ij} \gamma_j \sin (L - \theta_j).$$

In z' [formula (9)] these corrections cancel each other and it is thus superfluous to apply them.

The longitude v, as given by the theory, is longitude *in the orbit*. It should therefore be reduced to longitude on the fundamental plane by applying the correction

$$\delta v = -\tfrac{1}{4} i^2 \sin 2(v - \Omega).$$

This correction is very small, and can generally be neglected.

17. To get convenient units I divide the formulas (7) and (8) by $[a_i \text{ arc } 1^\circ]^2$, where a_i is the semiaxis major of the satellite's orbit. Everything is thereby expressed in jovicentric degrees (at mean distance) as units. In other words, in the computation of y and z I consider the radius vector to be expressed in a_i as unit (as it actually is in the quoted paper), and I replace $\sin \alpha$ by $\frac{\sin \alpha}{\text{arc } 1^\circ} = \alpha - (\alpha - \sin \alpha)^\circ$, and similarly for $\sin \beta$, $\tan B$, etc.

The two constants b and ρ, which are foreign to my theory, must be adopted from other sources. I adopt the following values

$$\log b = 6{\cdot}67896 - 10$$
$$\rho = 1/15.$$

This value of $\log b$, corresponding to $18''{\cdot}930$ at mean distance, was derived from Professor Sampson's observed maximum durations of eclipses, taking the mean of the four satellites.*

The value of ρ (which, of course, is the *geometrical* compression, not the *dynamical* one, which is used in the theory) was also derived from Professor Sampson's data. The observed durations give a determination of $z'(1 + c)$, where z' depends on the latitude and $(1 + c)$ on ρ. One has thus the choice of either adopting a value for ρ and correcting the latitudes, or adopting the latter and correcting ρ. Professor Sampson adopted the first course. To derive the best value for ρ to be used with my theory, I tried to find out what would have been the result if he had chosen the other alternative, assuming the latitudes as given by my theory to be correct. It is not feasible to derive the exact value of ρ which would have resulted from such a solution, without repeating a considerable part of Professor Sampson's solutions. But it is not difficult to find approximate values of ρ which will for each satellite make Professor Sampson's inclinations and nodes agree with mine. The results for the different satellites are rather discordant. I find

From satellite	I	$\delta\rho = -0{\cdot}011 \pm 0{\cdot}003$
,, ,,	II	$+0{\cdot}005 \pm {\cdot}0015$
,, ,,	III	$-0{\cdot}017 \pm {\cdot}003$
,, ,,	IV	$-0{\cdot}0065 \pm {\cdot}001$.

* In his Tables (which had not been published when this paper was written) Professor Sampson adopts $18''{\cdot}927$, and $\rho/1 + \rho = 1/15$, or $\rho = 1/14$.

The weighted mean $-0^{\circ}0045$ would give $\rho = 1/14^{\circ}94$, instead of which I adopted $\rho = 1/15$ as stated above.

With these values of b and ρ I find for C and $1+c$ the following values:—

$$C = C_0 \pm \delta C \qquad \begin{pmatrix} + \text{Ecl., Occ.} \\ - \text{Trans., Shad.} \end{pmatrix}$$

Sat.	C_0.	log Δ or log Rγ.	δC.	B or **B**.	log $(1+c)$.
	°		°		
I	9°7038			±0	0·02803
		0·60	0·0068		
II	6·0990			1	0·02795
		0·70	0·0054		
III	3·8235			2	0·02773
		0·80	0·0043		
IV	2·1739			3	0·02735
				4	0·02682

18. Let $y+dy$, $z'+dz'$, $c+dc$, $C+dC$ be the true values of the quantities of which the tabular values are y, z', c, C. Supposing the squares and products of dy, dz', . . . to be negligible, we find from every observation the following equation of condition for the determination of these corrections—

$$\frac{y(1+S^2)+SGz'(1+c)}{C}dy + \frac{z'(1+c)(1+G^2)+SGy}{C}[(1+c)dz'+z'dc] - dC + n = 0,$$

where n is defined by

$$2n = \frac{y^2+z'^2(1+c)^2+\{Sy+Gz'(1+c)\}^2}{C} - C$$

The divisor C has been introduced to get coefficients of convenient size. The equation as written down is rigorous (apart from the neglect of the square of dy, etc.), but in the coefficients (not, of course, in the computation of n) we can always neglect G^2, and nearly always SG, the equation thus becoming

$$\frac{y(1+S^2)}{C}dy + \frac{z'(1+c)}{C}[(1+c)dz'+z'dc] - dC + n = 0.$$

For eclipses and occultations, and for transits and shadow-transits without phase, we have, of course, rigorously $S = G = 0$. The form of the equation given here is only suggested as a convenient form for use. It is, of course, possible to write the equation in other forms which may be quite as good.

We have with sufficient accuracy

$$dC = C\frac{db}{b}$$

$$dc = d\rho$$

$$dy = dv - d\mathrm{L}$$

$$dz' = d\beta \pm d\mathrm{B} \qquad \begin{pmatrix} + \text{Ecl., Occ.} \\ - \text{Trans., Shad.} \end{pmatrix}$$

We can assume the coordinates of Jupiter as taken from the *N.A.* to be not in need of corrections. In that case we have

$$dL = 0$$
$$dB = \sin \lambda_{\mathcal{U}} d(\mathscr{I} \cos \mathscr{N}) - \cos \lambda_{\mathcal{U}} d(\mathscr{I} \sin \mathscr{N}).$$

For eclipses and shadow-transits $l_{\mathcal{U}}$ must be written for $\lambda_{\mathcal{U}}$.

The corrections dv and $d\beta$ to the longitude and latitude then furnish the corrections to the elements of the theory.

19. I will in this last section apply the formulas given above to an observation of a transit of satellite IV which was communicated to me by Mr. Innes, of Johannesburg. I quote from his letter, dated 1910 August 15:—"There was a partial transit of IV last night across the disc of Jupiter. At about $4^h\ 57^m$ G.T. one fifth of the disc of IV was on Jupiter. The discs were in contact from $4^h\ 46^m$ to $5^h\ 10^m$. Of course the times are somewhat uncertain with so slow a phenomenon, and where I say a fifth it might be $\frac{1}{4}$ to $\frac{1}{8}$ instead. But the observation appears to be interesting, as it is eight months since the last predicted transit, and one would have thought that IV was now quite clear of Jupiter." In fact, the last phenomenon of IV given in the *N.A.* is the shadow-transit on January 25, and we owe this important observation of an unpredicted phenomenon entirely to Mr. Innes's keenness.

We find from the *N.A.* for the time t_0 = Aug. 14, $4^h\ 57^m\ 0^s$

$$\odot \text{ (corrected)} = 141^\circ{\cdot}0080$$
$$\log R = 0{\cdot}005508$$
$$\log \Delta = 0{\cdot}780756.$$

The aberration-time, including the correction of art. 14, is found to be 3003^s. For the time $t = 4^h\ 6^m\ 57^s$ we then find

$$b_{\mathcal{U}} = +1^\circ{\cdot}2874$$
$$l_{\mathcal{U}} = 199{\cdot}9722$$
$$\log R_{\mathcal{U}} = 0{\cdot}736505.$$

From these we derive by the formulas (16)

$$\beta_{\mathcal{U}} = +1^\circ{\cdot}1627$$
$$\lambda_{\mathcal{U}} = 191{\cdot}7050,$$

and further by (17)

$$B = -2^\circ{\cdot}802 \qquad B = -2^\circ{\cdot}4318$$
$$\Lambda = 199{\cdot}915 \qquad L = 191{\cdot}6486.$$

The corrections for parallax (art. 15) have been neglected. They are

$$\delta B = -0^\circ{\cdot}00003 \qquad \delta L = -0^\circ{\cdot}00033.$$

The formulas (12) then give

$$\Psi = +8^\circ\cdot266 \qquad S = +0\cdot1451$$
$$B' - B = -0\cdot370 \qquad G = -0\cdot0073,$$

and with the arguments $\log\Delta$ and B we find from the tables of art. 17

$$C = 2\cdot1695 \qquad \log(1+c) = 0\cdot02760.$$

Further, I find from my theory of the satellites

$$\beta = -0^\circ\cdot3601$$
$$v = 371\cdot6733$$
$$r = 0\cdot99894.$$

The correction to reduce v to the fundamental plane (art. 16) does not reach the fourth decimal place. The common correction to reduce to the true equator (art. 16) has not been applied.

It is

$$\delta\beta = \delta B = \delta\mathbf{B} = +0^\circ\cdot0007.$$

Further, we find

$$\epsilon' = +0^\circ\cdot0057,$$

and therefore by (15) and (13)

$$y = +0\cdot0190 \qquad z = -0\cdot3597$$

and by (9)

$$z' = +2\cdot0699.$$

We are now in a position to derive the equation of condition. Such an equation, formed after the rules of art. 18, would, however, in this case be meaningless. The observer states that the centre of the satellite was projected outside the rim of Jupiter, and to compare this statement with theory we must compare the coordinates y and z' directly with the coordinates of the apparent edge of the planet's disc. As it is not our intention to derive corrections to the theory, but only to present clearly the theoretical circumstances of the observed phenomenon, it is preferable to express the coordinates in seconds of geocentric arc.* We find

$$1^\circ \text{ jovicentric} = 7''\cdot5133 \text{ geocentric.}$$

Therefore

$$y = +0''\cdot143 \qquad z' = +15''\cdot552.$$

* To be quite correct, the z coordinates should be multiplied by cos B $= 0\cdot99910$ to get the projection perpendicular to the line of sight. This has been omitted; the coordinates given are those in the plane $x = r'$.

The apparent semidiameters of Jupiter's disc are

$$C = 16''.300 \qquad \frac{C}{1+c} = 15''.296.$$

The edge of the apparent disc is given by the equation (8) for values of y algebraically smaller than $+0''.76$, and by (7) for the larger positive y's. We find

$y = 0$	$z' = +15''.295$		
$y = -0''.5$	$z' = +15''.285$	$y = +0''.5$	$z' = +15.289$
-1.0	$+15.258$	$+1.0$	$+15.268$
-1.5	$+15.216$	$+1.5$	$+15.231$
-2.0	$+15.161$	$+2.0$	$+15.181$

The satellite's coordinates are for the three epochs mentioned by Mr Innes,

$4^h\ 46^m$	$y = -1''.10$	$z' = +15''.544$
4 57	$+0.14$	$+15.552$
5 10	$+1.61$	$+15.562$

The uncertainty of these theoretical positions is, according to my judgment, not more than $\pm 0''.035$.

The distance of the satellite's centre from Jupiter's rim at $4^h\ 57^m$ was thus $0''.26$. According to Mr. Innes (*M.N.*, 1909, p. 525) the diameter is $1''.02$ at mean distance, or $0''.88$ at the actual distance at the time of observation. Consequently almost exactly one-fifth of the diameter was projected on the disc, which is in excellent agreement with Mr. Innes's observation.

If the coordinates given above are plotted, the curvature of the disc will be seen to be so slight that, if really so large a part of the diameter has been on the disc, the phenomenon must have lasted much longer than is stated by Mr. Innes. Thus either Mr. Innes's times of first or last contact are incorrect, or the minimum distance has been larger than was estimated by him. It should be remarked that this conclusion is entirely independent of the theory of the motion of the satellite. In neither case does this theory require any correction. If the second alternative is preferred, the adopted diameter of Jupiter should be slightly diminished, or the adopted ellipticity increased, or the adopted diameter of the satellite should be diminished.

It does not appear, however, that the evidence in favour of the second alternative is strong enough to warrant the derivation of such corrections.

Postscript.

In reply to a letter mentioning the results related in the last article, Mr. Innes writes on October 8 :—" I am in receipt of your

letters of the 9th and 10th ult., and you quite correctly point out an inconsistency in my observation of IV. My net result was: $\frac{1}{5}$ on; contacts-interval = 24 minutes. In weighting these, the contacts-interval should receive the greater weight. It might be lengthened to say 25 minutes. . . . I am very sure that it could not have exceeded 30 minutes, making every allowance for difficulty of observation."

Mr. Innes thus decidedly casts his vote in favour of the second of the alternatives mentioned above. Accordingly, I have derived an equation of condition on this assumption, introducing as unknowns only the corrections to the compression and the diameters of Jupiter and the satellite. To form this equation we have only to substitute for y and z' in the equation of art. 18 the coordinates of the point of contact. If y_0 and z'_0 are the coordinates of the centre of the satellite, we have

$$y = y_0 - \mathrm{R}\sin\alpha, \quad z' = z'_0 - \mathrm{R}\cos\alpha, \quad \tan\alpha = y_0/z'_0,$$

where R is the semidiameter of the satellite's disc.

We find for the two observed epochs—

4^h	46^m	$y_0 = -0\cdot1460$	$z_0' = +2\cdot0688$	$y = -0\cdot1419$	$z' = +2\cdot0103$
5	10	$+0\cdot2140$	$+2\cdot0712$	$+0\cdot2080$	$+2\cdot0129$

We have then

$$dy = -\sin\alpha\, d\mathrm{R} \qquad dz' = -\cos\alpha\, d\mathrm{R}.$$

The adopted value of R is $0\cdot0586 = 0''\cdot44$, as explained above. The equations of condition then become—

First contact: $\quad -0\cdot062\,\frac{d\mathrm{R}}{\mathrm{R}} - 2\cdot17\frac{db}{b} + 1\cdot93\,d\rho = 0\cdot0219 \pm 0\cdot0009$

Second „ : $\quad -0\cdot062^5$ „ $-2\cdot17$ „ $+1\cdot93$ „ $= 0\cdot0143 \pm 0\cdot0016$

The quantities added to the right-hand members with the sign ± are not meant as probable errors, but are the effects on these members of an alteration of one minute in the observed time. An increase of duration decreases the right-hand member.

Leiden:
1910 *October* 31.

Photographs of Halley's Comet made at the Córdoba Observatory, Argentine Republic.

(*Extract from a letter from Prof. C. D. Perrine.*)

I am sending to the Royal Astronomical Society some copies (enlarged nearly two times) on glass, of negatives of Halley's comet made here.

We secured a long series of good photographs of the comet during its greatest brightness, of which the following is a summary:—

Position plates with the astrographic telescope:

April 18–30	9 nights.
May	22 ,,
June	23 ,,
July	14 ,,
August	3 ,,

Long and short exposures for structure were obtained on practically the same nights to and including the first week in July, after which the comet was too faint to justify other than position plates.

An equally extensive series was obtained with the portrait lens of 5-inch aperture.

Extra-focal observations were also obtained on 40 nights for determinations of brightness.

Observations were secured with two small objective prism spectrographs on 40 nights between May 21 and July 28. One of the spectrograms recorded the spectrum of the dislocation in the tail observed on June 6. A casual examination shows no difference of constitution between this disturbance and the normal tail of the night before.

Photographs of this disturbance were obtained on June 6, 7 and 8, which will give good values of the velocities of recession from the head.

A small camera was improvised to record the great lengths of tail visible before the transit in May.

Micrometer observations were begun on 1909 November 30, and obtained on 46 nights between that date and 1910 February 3, inclusive, when the comet was getting too low in the west to allow of as accurate observations as could be made in the northern hemisphere. A few observations were obtained after the comet became visible in the morning sky in April, and were resumed again on August 6. Positions were obtained on 10 nights from August 6 to 22 inclusive. After the latter date it was too low in the west for satisfactory observations. The comet was seen, however, on August 25.

It is clear that the bright streamer of the tail was not

encountered by the Earth, but passed to the north of us. There was, however, a wide, faint streamer to the south which was observed here on the mornings of May 19 and 21 (civil time), (May 20 was cloudy), which, if it persisted, should have encountered the Earth. On the morning of May 18 (civil time) a mistiness was observed to the east, which may possibly have been a faint tail streamer near us. If it was in reality a streamer, it was probably nearly in the direction of the radius-vector, and the Earth would have passed through it at or near the time of transit.

It is certain that the main tails lagged considerably behind the radius-vector.

I notice the suggestion of Innes that the Earth may have the power of repelling cometary tail-matter in a similar way to the Sun. It seems to me doubtful if this is the case. If this force is in reality light pressure, as seems very probable, the small intensity of the sunlight reflected by the Earth could hardly overbalance its gravitational force. If the force of repulsion is electric or of some other nature, then the case is otherwise.

The comet was a magnificent spectacle, especially during the week before the transit. That great searchlight beam, extending away from the Sun for 120° to 130°, was a sight which cannot be forgotten.

Córdoba:
1910 *September* 6.